

UDC 621.3:[681.5+681.6]

DOI: 10.25140/2411-5363-2019-4(18)-148-154

Erik Prada

## DETERMINATION OF TRANSFER FUNCTION OF MAGNETIC LEVITATION MODEL AND EXPERIMENTAL VERIFICATION OF OPTICAL SENSOR

**Urgency of the research.** The potential of controlling the position of levitating objects has great application in deposition and in various positioning systems. Magnetic levitation eliminates direct mechanical friction between moving parts.

**Target setting.** The measurement shielding method used is one of the methods of determining the position of a levitating object. By combining positioning and regulating elements, we achieve a feedback control. The use of a given type of measurement has advantages in places where the use of other methods is not appropriate.

**Actual scientific researches and issues analysis.** The problem of magnetic levitation is addressed by several research laboratories with a direct connection to practice. The problem that is currently solved within magnetic levitation is the regulation of the levitating object using various types of regulators.

**The research objective.** Derivation of mathematical model of magnetic levitation and examination of nonlinear system followed by linearization by Taylor series. Experimental determination of characteristics and dependence between object position, voltage and current.

**The statement of basic materials.** The position of the levitating object is determined by the shading of the optical sensor. The light source is a laser light.

**Conclusions.** In this work we defined the mathematical model of the magnetic levitation system and subsequently derived the transfer function of the levitation system and the position sensor. From the experimental verification of the shadow method for the determination of the position of the levitating object and the consequent need for regulation, we found that the dependence of the position of the levitating object on current and voltage on the photodiode is linear in the active region.

**Keywords:** magnetic levitation; optical sensor; laser module; motion detection.

Fig.: 9. References: 11.

**Introduction.** Magnetic levitation has a large perspective in practice, but the widespread use of this technology is not as enormous as some other technologies. The best-known application of magnetic levitation is the use of maglev trains, but it is not the only application of magnetic levitation in practice. To meet the functional model of magnetic levitation in practice is quite problematic in our latitudes. Germany is one of the few countries dedicated to magnetic levitation technology and has a high reputation worldwide with its Transrapid train. Another application of the use of magnetic levitation is, for example, in the Trimble® S6 servo system Fig. 1.

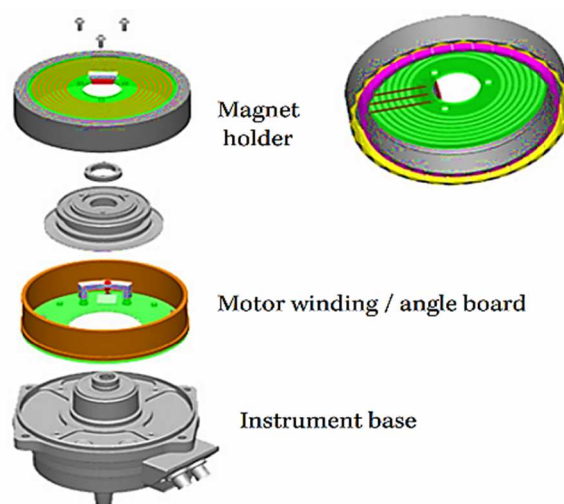


Fig. 1. Trimble S6 (integrated angle and servo system) [1]

Trimble® MagDrive™ servo technology is an integrated servo and angle system that uses a direct drive and frictionless electromagnetic drive technique similar to those used in maglev trains. The direct drive system allows the servo motors to be mounted directly on the horizontal and vertical axis, removing the need for additional mechanical gearing [1].

**Shadow method of measuring position.** Magnetic levitation has a large perspective in practice, but the widespread use of this technology is not as enormous as some other technologies. The measurement of the position of the levitating object using the shadow method is based on the measurement of the current depending on the intensity of the incident light beam on the photosensitive sensor. The drop shadow on the photodiode will cause us to drop the current. Classic light or intense laser light can be used as the light beam source. The sensing unit thus consists of an emitter and an emitted beam sensor. It is most ideal to use a laser beam source as the emitter, the intensity of which is better reflected in the photodiode in a way of greater variance of the measured values. The figure Fig. 2 shows a diagram of the construction of the sensing. When designing it is appropriate to use a collimator, which provides us collimated beam [2-5].

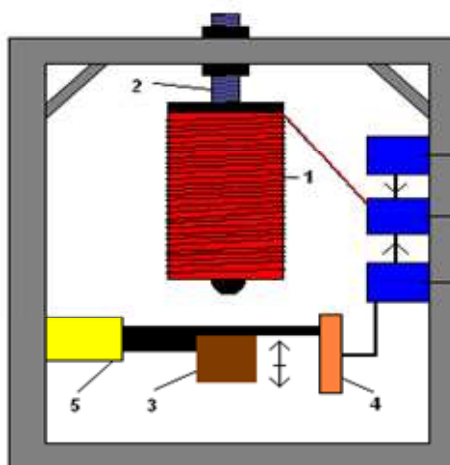


Fig. 2. Schematic representation of the position sensing solution by the shadow method:  
1 – coil, 2 – connecting screw, 3 – levitating object, 4 – fotodiode, 5 – laser module with collimator

**Mathematical model of the levitating system.** In order to be able to regulate this dynamic system, we describe the physical behavior of the magnetic-levitation dynamic system by a mathematical model. Principle of behavior of levitating body, description of its movement and realization of mathematical model is in following lines.

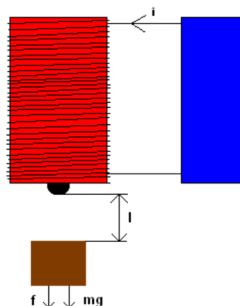


Fig. 3. Forces acting on a levitating object

The electromagnet inductance is assumed in the form:

$$L(y) = L_1 + \frac{L_0}{(1+\frac{y}{a})} \tag{1}$$

We also assume that there is a relation between the inductance constants:

$$L_1 \gg L_0, y(0) = 1 \tag{2}$$

The electromagnetic force acting on a levitating object can be expressed from the magnetic energy equation:

$$W'(i, y) = \frac{1}{2} i^2 L(y) \tag{3}$$

Electromagnetic force is defined as a derivative of magnetic energy and after substitution of induction of electromagnet and energy of magnetic field we get:

$$F_m(i, y) = \frac{\partial W'}{\partial y} = -\frac{L_0 i^2}{2a\left(1+\frac{y}{a}\right)^2} \quad (4)$$

In the next step, we can derive the equation of motion of the levitating object from Fig. 3:

$$m\ddot{y} = mg + F_m \quad (5)$$

Substituting Equation (4) into equation (5) gives a nonlinear motion equation:

$$m\ddot{y} = mg - \frac{L_0 i^2}{2a\left(1+\frac{y}{a}\right)^2} \quad (6)$$

As the role of the electromagnetic actuator is generating such a magnetic force to be balanced against the gravitational force is necessary to equation (6) adjust into static equilibrium:

$$mg = \frac{L_0 i^2}{2a\left(1+\frac{y}{a}\right)^2} \quad (7)$$

In the next step, it is necessary to modify equation (7) to express the current:

$$I = \left(1 + \frac{Y}{a}\right) \sqrt{\frac{2mga}{L_0}} \quad (8)$$

Nonlinear equation of motion (6) can be developed into Taylor series. Then we get:

$$F_m(\Delta i, \Delta y) = F_m(I, Y) + \frac{\partial F_m(I, Y)}{\partial y} \Delta y + \frac{\partial F_m(I, Y)}{\partial i} \Delta i \quad (9)$$

Where the fault variables are equal:  $\Delta y = y - Y$ ,  $\Delta i = i - I$ .

After substituting force equations into equation (9) we get:

$$F_m(\Delta i, \Delta y) = -\frac{L_0 I^2}{2a\left(1+\frac{Y}{a}\right)^2} + \frac{L_0 I^2}{a^2\left(1+\frac{Y}{a}\right)^3} \Delta y - \frac{L_0 I}{a\left(1+\frac{Y}{a}\right)^2} \Delta i \quad (10)$$

After modification, the basic linearized equation has the form:

$$m\Delta\ddot{y} = mg + F_m(\Delta i, \Delta y) \quad (11)$$

Substituting equation (10) into equation (11) and then simplifying the relations we get:

$$m\Delta\ddot{y} = \frac{L_0 I^2}{a^2\left(1+\frac{Y}{a}\right)^3} \Delta y - \frac{L_0 I}{a\left(1+\frac{Y}{a}\right)^2} \Delta i \quad (12)$$

In the next step, it is necessary to simplify equation (12) by multiplying both expressions on the right and left sides:

$$\frac{2mga\left(1+\frac{Y}{a}\right)^2}{L_0 I^2} = 1 \quad (13)$$

The resulting equation will then have the form:

$$\Delta\ddot{y} - \left(\frac{2g}{a+Y}\right) \Delta y + \left(\frac{2g}{I}\right) \Delta i = 0 \quad (14)$$

Using the Laplace transform we get the following expression:

$$s^2 \Delta Y(s) - \left(\frac{2g}{a+Y}\right) \Delta Y(s) + \left(\frac{2g}{I}\right) \Delta I \quad (15)$$

By adjusting equation (15) we get the transfer function:

$$\frac{\Delta Y(s)}{\Delta I(s)} = -\frac{1}{As^2 - B} \quad (16)$$

Where the individual coefficients A, B:

$$A = -\frac{I}{2g} \left[\frac{As^2}{m}\right], B = \frac{I}{a+Y} \left[\frac{A}{m}\right] \quad (17)$$

Since in expression (17) the coefficient  $B$  is negative and the coefficient at the first derivative missing, this system is unstable. It is therefore necessary to ensure system stability feedback. We need numerical values for the electrical and mechanical subsystem directly to models [2-5].

**Experimental verification of the shadow method.** The absolute measurement method was used for experimental verification. The aim of the measurement was experimental verification of the proposed solution. The experiment was performed under different conditions and settings and was therefore divided into several phases. The determined dependency characteristic is therefore different for each phase.

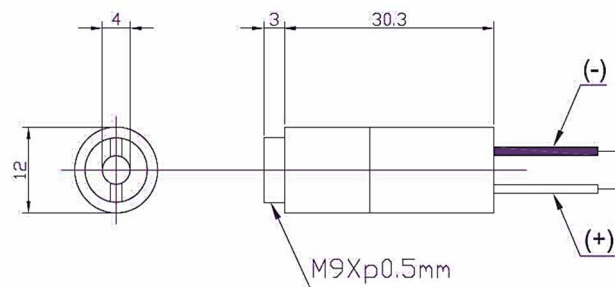


Fig. 4. Laser diode HLDPM12 – 655 – 5

In the experiment we used Laser diode HLDPM12 – 655 – 5 in Fig. 4 with collimator, and Tesla 1PP75 photodiode. The active surface of the Tesla 1PP75 photodiode is 3.5 mm x 5.5 mm. Current measurement was performed on a HP 34401A professional laboratory multimeter. The casting of the shadow on the photodiode was obtained using a metal sheet that was mounted in a rack with micrometer movement in the X-axis and Y-axis directions. Schematic representation of the measurement is in the figure Fig. 5.

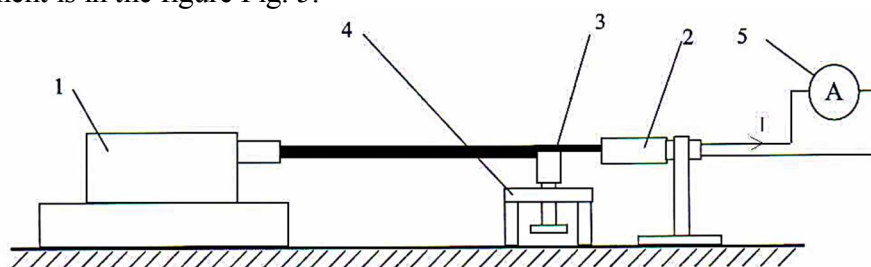


Fig. 5. Scheme of current measurement by laser beam on photodiode:

1 – Laser module, 2 – photodiode, 3 – Metal shielding plate,  
4 – Stand with micrometer movement, 5 – ammeter

Measurement procedure:

- connecting the laser to the mains, attaching the laser probe to the stand and connecting the photodiode through the wires to the multimeter input;
- turning on the laser and checking the beam so that it hits the sensor;
- setting the distance of the sensor from the laser module as required;
- grasping the shielding plate in a micrometer feed rack;
- zero setting of the shielding plate;
- turning on the multimeter and setting the DC current mode;
- recording the generated background currents of the measuring room;
- reading the value from the multimeter with zero cover;
- turn the screw to change the position of the shielding plate in 0.5 mm increments until the entire 10 mm interval has passed;
- reading three values from the multimeter every half millimeter and writing to the table;
- calculation of averages from the measured values and subsequent correction for total measurement error;
- Interpolation graphs [5].

*Laser diode HLDPM12 – 655 – 5 parameters*

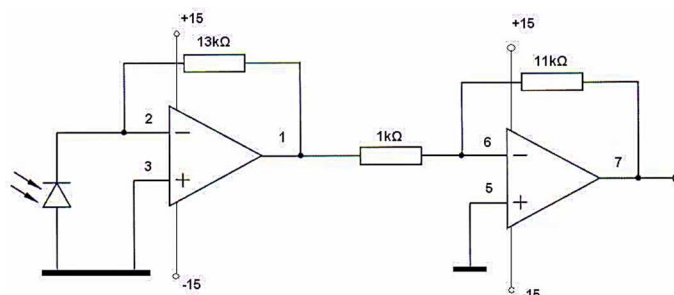
Laser diode	Supply voltage [V]	Wavelength[nm]	Output voltage [mV]
HLDPM12 – 655 – 5	2,5 ~ 4,0	655	6,0 ~ 7,0



*Fig. 6. Model of design of collimator and laser diode*

If light is incident on the diode, there is an electrical voltage on the diode leads. From the characteristic of the photodiode we know that the dependence of the voltage on the illumination is largely non-linear, while the dependence of the current on the illumination is linear. For this reason it is advantageous to connect the photodiode to the current-voltage converter.

The current is applied to the inverting input of the operational amplifier. Since the input resistance of the amplifier is large, it also passes through resistor R2 in the feedback of the operational amplifier. Due to the high gain, the feedback tries to keep the input close to the potential of the non-inverting input. The input voltage of the converter U1 is close to zero as well as the input resistance. The output voltage of the circuit represents the voltage drop induced by the current I on the resistor R2. The resistance of resistor R2 must be chosen so that the output voltage is in the linear range. If the operational amplifier output becomes saturated, the wiring does not work. Since the amplification of the converter was not sufficient, another amplification stage implemented by the operational amplifier TL072 was used.



*Fig. 7. Schematic of current-voltage converter*

The dependence of the output current on the position of the levitating object is :

$$I = a l + b \tag{18}$$

Where the coefficients a, b of equation (18) can be determined using the least squares method.

$$a = \frac{\sum_{i=1}^{14} l_i \cdot \sum_{i=1}^{14} l_i^2 - \sum_{i=1}^{14} l_i \cdot \sum_{i=1}^{14} I_i l_i}{14 \cdot \sum_{i=1}^{14} l_i^2 - \left(\sum_{i=1}^{14} l_i\right)^2} \tag{19}$$

$$b = \frac{\sum_{i=1}^{14} I_i l_i - \left(\sum_{i=1}^{14} l_i\right) \cdot \left(\sum_{i=1}^{14} I_i\right)}{14 \cdot \sum_{i=1}^{14} l_i^2 - \left(\sum_{i=1}^{14} l_i\right)^2} \tag{20}$$

By substituting the measured values into equations we get the resulting coefficients a, a b:

$$I = 14,45l - 2,12 \tag{21}$$

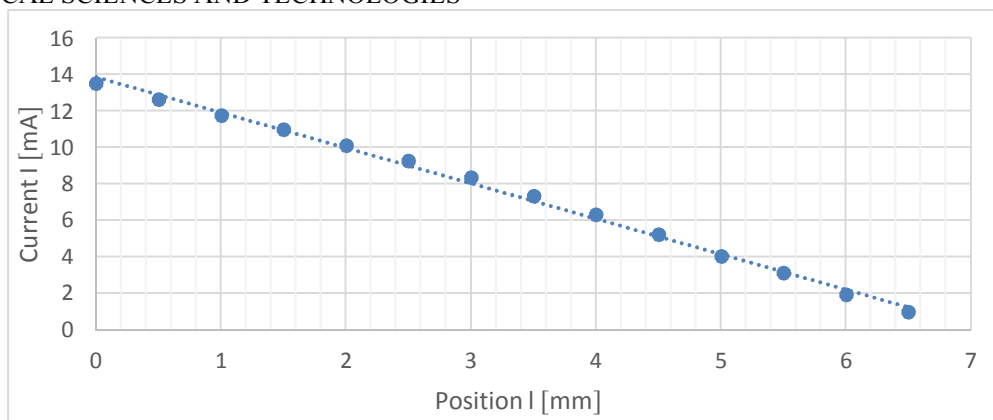


Fig. 8. Graphical representation of current versus distance

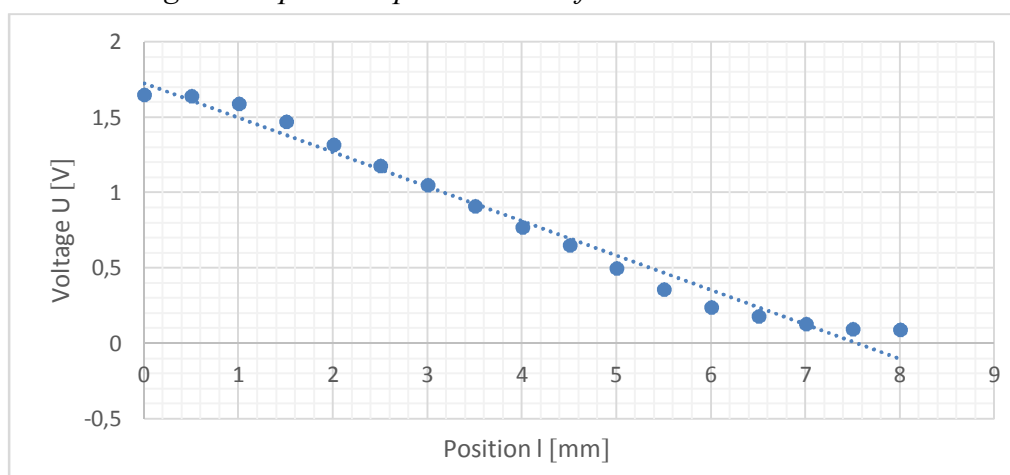


Fig. 9. Graphical representation of voltage versus distance

From the measured data it can be seen that the working range of the sensor is 6.5 mm, while the dependence is linear. Transfer functions of the sensor have the following form:

$$G_{sn} = \frac{\Delta I}{\Delta l} \tag{22}$$

$$G_{sn} = \frac{\Delta U}{\Delta l} \tag{23}$$

**Conclusions.** In this work we defined the mathematical model of the magnetic levitation system and subsequently derived the transfer function of the levitation system and the position sensor. From the experimental verification of the shadow method for the determination of the position of the levitating object and the consequent need for regulation, we found that the dependence of the position of the levitating object on current and voltage on the photodiode is linear in the active region [1-11].

**Acknowledgement.** The authors would like to thank to Slovak Grant Agency – project KEGA 018TUKE-4/2018 “Implementation of new technologies and educational methods in the field of control systems to improve education level and practical skills of graduates of Mechatronics” and project VEGA 1/0389/18, “Research and development of kinematic redundant mechanisms”.

### References

1. LEMMON, T., JUNG, R., (2005). Trimble S6 with MagDrive servo technology. White Paper. Trimble Survey, Westminster, Colorado, USA. Online <http://www.geotechmabna.com/catalogues/Papers/2total/4.pdf>.
2. Reich, Štefan. (2003). Návrh funkčného modelu magnetickej levitácie: Diplomová práca. Košice : TU – Sjf, 2003.

3. Prada, Erik. (2009) Bezdotyková snímacia jednotka polohy levitujúceho objektu: Bakalárska práca. Košice : TU – SjF, 2009.
4. Valašteková, Radka. (2009). Návrh snímača polohy a akčného člena pre didaktický model magnetickej levitácie: Diplomová práca. Košice : TU – SjF.
5. Prada, E., Baločková, L., Surovec, R. (2012). Experimentálne overenie snímača polohy levitujúceho objektu. II. VEDECKÁ KONFERENCIA DOKTORANDOV, KOŠICE, 2012.
6. Gmíterko, A., Kelemen, M., Dovica, M., Capák, M. (1999). Miniature mobile robot for moving in a tube with small diameter. Proceedings of the 2nd International Conference Mechatronics and Robotics '99, Brno, Czech Rep., 1999, 67-70, ISBN 80-214-0604-6.
7. VITKO, A., JURIŠICA, L., BABINEC, A., DUCHOŇ, F., KEÚČIK, M. (2010). Some Didactic Problems of Teaching Robotics. Proceedings of the 1st International Conference Robotics in Education 2010. Bratislava, 16.-17. 9. 2010, Bratislava, Slovak University of Technology in Bratislava, ISBN 978-80-227-3353-3, 27-30.
8. Havlík, Š., Hricko, J., Prada, E., & Jezný, J. (2020). Linear motion mechanisms for fine position adjustment of heavy weight platforms doi:10.1007/978-3-030-19648-6\_3.
9. I. Virgala et al., "Positioning of Pneumatic Actuator Using Open-Loop System", Applied Mechanics and Materials, Vol. 816, pp. 160-164, 2015.
10. E. Prada et al., "Simulation and Determination of the Influence of the Gait Function on the Change of the Shape of a Snake-Like Robot", Applied Mechanics and Materials, Vols. 789-790, pp. 636-642, 2015.
11. SPANIKOVA, G., SPANIK, P., FRIVALDSKY, M. et al. (2017). Electric model of liver tissue for investigation of electrosurgical impacts. Electrical Engineering, 99 (4), 1185-1194. doi.org/10.1007/s00202-017-0625-0.

UDC 621.3:[681.5+681.6]

*Ерік Прада*

## ВИЗНАЧЕННЯ ПЕРЕДАВАЛЬНОЇ ФУНКЦІЇ ДЛЯ МОДЕЛІ МАГНІТНОЇ ЛЕВІТАЦІЇ ТА ЕКСПЕРИМЕНТАЛЬНА ПЕРЕВІРКА ОПТИЧНОГО ДАТЧИКА

*Актуальність теми дослідження.* Керування положенням об'єктів, що левітують, має великий потенціал в застосуваннях, пов'язаних з осажденнями та в різноманітних системах позиціонування. Магнітна левітація виключає пряме механічне зчеплення між рухомими частинами.

*Постановка проблеми.* Використовуваний метод екранування вимірювань – один із методів визначення положення об'єкта, що левітує. Шляхом комбінування елементів позиціонування та регулювання, досягається контроль зворотного зв'язку. Використання заданого типу вимірювання має переваги в місцях, де використання інших методів не є доцільним.

*Аналіз останніх досліджень і публікацій.* Проблема магнітної левітації вирішується багатьма науково-дослідними лабораторіями, які мають прямий зв'язок із практичними дослідженнями. Проблема, яка в даний час вирішується в області магнітної левітації, – це регулювання стану об'єкта, що левітує, за допомогою різних типів регуляторів.

*Постановка завдання.* Виведення математичної моделі магнітної левітації та дослідження нелінійної системи з наступною лінеаризацією за допомогою ряду Тейлора. Експериментальне визначення характеристик та залежностей між положенням об'єкта, та керуючими напругою та струмом.

*Виклад основного матеріалу.* Положення об'єкта, що левітує, визначається шляхом затінення оптичного датчика. Джерелом світла виступає лазер.

*Висновки відповідно до статті.* У даній роботі визначено математичну модель системи магнітної левітації, що згодом дозволило отримати передавальну функцію системи левітації та датчика положення. З експериментальної перевірки методом затінення для визначення положення об'єкта, що левітує, та відповідно до цього положення, необхідності в регулюванні, було встановлено, що залежність положення об'єкта, що левітує, від струму та напруги на фотодіоді є лінійною в активній області.

**Ключові слова:** магнітна левітація; оптичний датчик; лазерний модуль; детектування руху.

Рис.: 9. Бібл.: 11.

**Erik Prada** – Doctor of Technical Sciences, Research Fellow, Technical University of Kosice Department of Mechatronics, Faculty of Mechanical Engineering (Park Komenského 8, 04200 Košice, Slovak Republic).

**Ерік Прада** – дослідник, кандидат технічних наук, Технічний університет Кошице (Park Komenského 8, 04200 Košice, Slovak Republic).

**E-mail:** erik.prada@tuke.sk

**ORCID:** <https://orcid.org/0000-0001-7226-1240>

**ResearcherID:** R-3524-2016

**Scopus Author ID:** 55873860000