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ACTIVE DAMPING OF NONSTATIONARY VIBRATIONS IN A BEAM WITH ELECTRO-VISCOELASTIC PATCHES

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АКТИВНЕ ДЕМПФІРУВАННЯ НЕСТАЦІОНАРНИХ КОЛИВАНЬ БАЛКИ З ЕЛЕКТРОВ'ЯЗКОПРУЖНИМИ НАКЛАДКАМИ

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АКТИВНОЕ ДЕМПФИРОВАНИЕ НЕСТАЦИОНАРНЫХ КОЛЕБАНИЙ БАЛКИ С ЭЛЕКТРОВЯЗКОУПРУГИМИ НАКЛАДКАМИ

The technique of finite element modeling in integral Fourier transform frequency space is applied to the problem of active damping of nonstationary vibration in a beam with piezoelectric patches. The optimal value of a feedback control parameter by maximum damping criterion is found.

Key words: *non-stationary vibrations, active damping, smart constructions, Fourier transformation*.

Для вирішення завдання активного демпфірування нестаціонарних коливань балки з п'єзоелектричними накладками застосовано методику скінченно-елементного моделювання у частотному просторі інтегральних перетворень Фур'є. Знайдено оптимальне значення керуючого параметра зворотного зв'язку за критерієм максимального демпфірування.

Ключові слова: *нестаціонарні коливання, активне демпфірування, smart-конструкція, перетворення Фур'є.*

Для решения задачи активного демпфирования нестационарных колебаний балки с пьезоэлектрическими накладками применена методика конечно-элементного моделирования в частотном пространстве интегральных преобразований Фурье. Проведен поиск оптимального значения параметра обратной связи по критерию максимального демпфирования.

Ключевые слова: *нестационарные колебания, активное демпфирование, smart-конструкция, преобразование Фурье.*

Introduction. The passive damping of nonstationary vibration in a beam with electroviscoelastic dissipative patches was studied in [1]. The application of piezoelectric elements with connected electric circuits (RL-shunts) was shown to decrease the reaction of structures to the impact of nonstationary loads. Dissipation of the vibration energy is conducted at the electropassive elements (sensors) and RL-shunts due to conversion of electric energy into heat.

In recent years the methods of active damping using active and passive piezoelectric elements – sensors and actuators [2–7], became intensively employed for damping of vibration in thin-shelled structure elements. Different modes of applying voltage to the active elements to effect the structure are being used. In particular, the calculated beforehand potential difference is applied using a control device to the actuator, to compensate vibration on a specific frequency [5–7].

The adjusting of the potential difference on the electrodes of the actuator φ_a can be done considering the indications of sensor φ_s. In fig. 1 the schematic circuit of connecting a sensor and an actuator that have reverse polarization in a circuit of negative feedback is shown. Aside from this scheme, an RL-shunt is connected to the passive piezoelectric elements of the viscoelastic material PVDF.

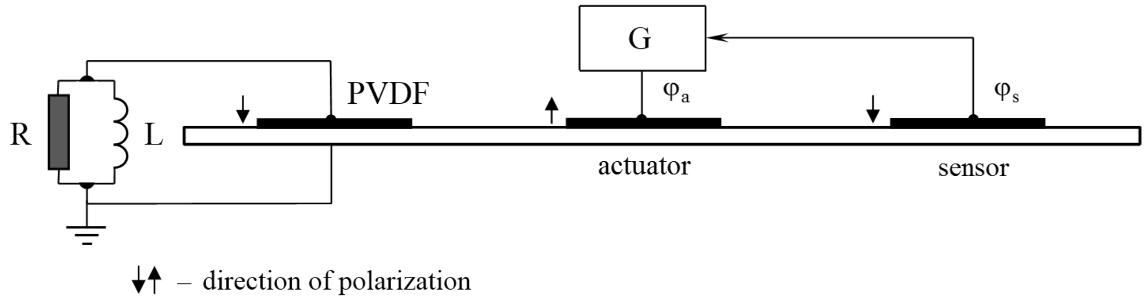


Fig. 1. Schematic circuit of active damping of a structure fragment

Different algorithms of feedback are considered in [6]. In this paper the actuator potential is governed by the rule:

$$\varphi_a = -G\dot{\varphi}_s, \qquad (1)$$

where G is a coefficient of amplifying the derivative of a sensor potential by time (control parameter). This coefficient influences the inertial and dissipative characteristics of a beam.

To efficiently implement the active control of vibration, a particularly important issue is to consider the dissipation of energy by methods that are based on real physical dependencies for structure materials and piezoelectric materials. The existing papers on vibration control mostly omit these dependencies for several reasons. However, the experiments [8] prove that passive and active piezoelectric elements under dynamic loads behave as viscoelastic materials. It is shown in [9; 10] that the analysis of nonstationary vibration in imperfectly elastic structures can be efficiently conducted by the frequency finite element method (FFEM) [11], which performs the synthesis of structures and the analysis of vibration in Fourier integral transform space. The advantages of this method are the capability to take into account the dependencies of the linear theory of hereditary environments, including the correct introduction of frequency-dependent complex modules, and the capability to analyze nonstationary vibration with given initial conditions. The conversion to frequency space also significantly simplifies the synthesis of structures from piezoelectric materials.

Mathematical model. The complexity of problems requires the application of approximate methods. The synthesis of the finite element beam model with passive electroviscoelastic patches was conducted by the FFEM [1]. In case of active vibration control, the finite element version of differential equations for balance and quasi-static equations of linear electroelasticity is supplemented with the equations for the sensor (s) and the actuator (a). The dynamics equations, received by the application of variational Hamilton-Ostrohradsky principle, after the integral Fourier transform will look like the equations of the linear elasticity theory with complex modules:

$$\begin{aligned} &(-\omega^2)\mathbf{M}\tilde{\mathbf{u}} + \tilde{\mathbf{K}}_u\tilde{\mathbf{u}} + \tilde{\mathbf{K}}^s_{u\varphi}\tilde{\varphi}_s + \tilde{\mathbf{K}}^a_{u\varphi}\tilde{\varphi}_a = \tilde{\mathbf{F}}(\omega) + \mathbf{f}, \\ &\tilde{\mathbf{K}}^{s\,T}_{u\varphi}\tilde{\mathbf{u}} - \tilde{\mathbf{K}}^s_{\varphi}\tilde{\varphi}_s = \tilde{\mathbf{Q}}_s(\omega), \\ &\tilde{\mathbf{K}}^{a\,T}_{u\varphi}\tilde{\mathbf{u}} + \tilde{\mathbf{K}}^a_{\varphi}\tilde{\varphi}_a = \tilde{\mathbf{Q}}_a(\omega), \end{aligned} \qquad (2)$$

where $\mathbf{M}$ and $\tilde{\mathbf{K}}^{s(a)}_{\varphi}$ are matrices of mass and electric "stiffness" of sensor (actuator) respectively, $\tilde{\mathbf{K}}^{s(a)}_{u\varphi}$ and $\tilde{\mathbf{K}}^{s(a)}_{\varphi u}(\omega) = \tilde{\mathbf{K}}^{s(a)T}_{u\varphi}(\omega)$ are matrices that correspond to the direct and reverse piezoelectric effect respectively:

$$\mathbf{M} = h\int_0^a\int_0^b \mathbf{N}_u^T \rho \mathbf{N}_u \, dydx, \; \tilde{\mathbf{K}}_u(\omega) = h\int_0^a\int_0^b (\mathbf{A}\mathbf{N}_u)^T \tilde{\mathbf{C}}(\omega)(\mathbf{A}\mathbf{N}_u)\, dydx,$$

$$\tilde{\mathbf{K}}_{u\varphi}^{s(a)}(\omega) = h\int_0^a\int_0^b (\mathbf{A}\mathbf{N}_u)^T \tilde{\mathbf{e}}(\omega)^T \nabla\mathbf{N}_\varphi dydx, \quad \tilde{\mathbf{K}}_{\varphi}^{s(a)}(\omega) = h\int_0^a\int_0^b (\nabla\mathbf{N}_\varphi)^T \tilde{\mathbf{\kappa}}(\omega)\nabla\mathbf{N}_\varphi dydx, \quad (3)$$

$$\tilde{\mathbf{F}}(\omega) = \int_0^\infty\int_0^a\int_0^b \nabla\mathbf{N}_u^T \mathbf{p}(x,y,t)\exp(-i\omega t)dydxdt,$$

$$\tilde{\mathbf{Q}}_{s(a)}(\omega) = \int_0^\infty\int_0^a\int_0^b \nabla\mathbf{N}_\varphi^T \mathbf{q}(x,y,t)\exp(-i\omega t)dydxdt,$$

where $\tilde{\mathbf{C}}(\omega) = \mathbf{C}'(\omega) + i\mathbf{C}''(\omega)$ is the matrix of frequency-dependent complex elastic modules; $\tilde{\mathbf{e}}(\omega) = \mathbf{e}'(\omega) + i\mathbf{e}''(\omega)$, $\tilde{\mathbf{\kappa}}(\omega) = \mathbf{\kappa}'(\omega) + i\mathbf{\kappa}''(\omega)$ are the matrices of complex piezoelectric and dielectric modules respectively; ρ is material density; h is the width of a finite element; $\mathbf{p}(x,y,t) = (p_x \ p_y)^T$ is external load $\mathbf{q}(x,y,t) = (q_x \ q_y)^T$; $\tilde{\mathbf{F}}(x,y)$ is the Fourier image of external mechanical load; $\tilde{\mathbf{Q}}_{s(a)}(x,y)$ is the image of nodal charges vector; $\tilde{\mathbf{u}}$, $\tilde{\varphi}_{s(a)}$ are the images of mechanical displacements and potentials of a sensor (actuator) in Fourier space respectively; $\mathbf{f} = i\omega\mathbf{M}\dot{\mathbf{u}}(0) + \mathbf{M}\mathbf{u}(0)$, where $\dot{\mathbf{u}}(0)$, $\mathbf{u}(0)$ are the initial velocities and displacements of nodal points respectively; $i = \sqrt{-1}$.

The application of Hamilton-Ostrohradsky variational principle gives the approximate result [12]. The more correct transition method, based on variational equations in convolutions, equations of hereditary viscoelasticity theory and further Fourier transform, is applied in papers [10; 13].

In frequency space the equation (1) becomes:

$$\tilde{\varphi}_a = -i\omega G\tilde{\varphi}_s. \quad (4)$$

The difference of potentials, necessary to apply to the actuator electrodes, is determined from the system (2):

$$\tilde{\varphi}_a = -i\omega G\tilde{\mathbf{K}}_\varphi^{s\,-1}\tilde{\mathbf{K}}_{u\varphi}^{s\,T}\tilde{\mathbf{u}}. \quad (5)$$

Thus we get an equation in respect to images of mechanical displacements:

$$\tilde{\mathbf{Z}}(\omega)\tilde{\mathbf{u}} = \tilde{\mathbf{F}}(\omega), \quad (6)$$

where $\tilde{\mathbf{Z}}(\omega)$ is a dynamic stiffness matrix

$$\tilde{\mathbf{Z}}(\omega) = -\omega^2\mathbf{M} + \tilde{\mathbf{K}}_u + \tilde{\mathbf{K}}_{u\varphi}^s(\tilde{\mathbf{Q}}_s(\omega) + \tilde{\mathbf{K}}_\varphi^s)^{-1}\tilde{\mathbf{K}}_{u\varphi}^{s\,T} - i\omega\tilde{\mathbf{K}}_{u\varphi}^a G\tilde{\mathbf{K}}_\varphi^{s\,-1}\tilde{\mathbf{K}}_{u\varphi}^{s\,T}. \quad (7)$$

The solution of the linear algebraic system of equations (2) in respect to displacements in frequency space can be written as:

$$\tilde{\mathbf{u}} = \tilde{\mathbf{Z}}(\omega)^{-1}\tilde{\mathbf{F}}(\omega). \quad (8)$$

After we determine the displacements, the return to the time space is done by the discrete reverse Fourier transform, namely the fast Fourier transform (FFT):

$$\mathbf{u} = \mathrm{FFT}^{-1}(\tilde{\mathbf{Z}}(\omega)^{-1}\tilde{\mathbf{F}}). \quad (9)$$

To analyze the dissipation of vibration energy in a structure, the eigenvectors and eigenvalues of a dynamic stiffness matrix need to be determined from the requirement

$$\left|\tilde{\mathbf{Z}}(\omega)\right| = 0. \quad (10)$$

The decrement is defined by the formula:

$$\Delta = 2\pi \frac{\omega_k''}{\omega_k'}, \tag{11}$$

where ω_k'', ω_k' are the real and imaginary frequencies on the k-th vibration form respectively.

The calculation for a beam with active piezoelectric patches. Let's consider an example of calculating the nonstationary vibration of a beam [1] with active viscoelastic piezoelectric patches.

The beam parameters:

– length $l = 0,7\ m$;

– width $b = 20 \cdot 10^{-3}\ m$;

– the main bearing structure thickness $s = 6 \cdot 10^{-3}\ m$;

– material density $\rho = 2,7 \cdot 10^3\ \frac{kg}{m^3}$;

– the elasticity module of the bearing layer $E = 6,71 \cdot 10^{10} \cdot (1 + i \cdot 0,025)\ Pa$.

The patch parameters of the viscoelasic piezocomposite material based on PVDF and PZT:

– patch length $l_p = 0,1 m$;

– width $b_p = 20 \cdot 10^{-3} m$;

– thickness $s_p = 3 \cdot 10^{-3} m$;

– material density $\rho_p = 1,75 \cdot 10^3\ kg/m^3$;

– real and imaginary components of the elasticity modules matrix for the piezoelectric material: $C_{11} = 15,7 \cdot 10^9 \cdot (1 + i \cdot 0,064)\ Pa$, $C_{31} = 9,30 \cdot 10^9 \cdot (1 + i \cdot 0,098)\ Pa$, $C_{33} = 13,6 \cdot 10^9 \cdot (1 + i \cdot 0,069)\ Pa$, $C_{55} = 2,52 \cdot 10^9 \cdot (1 + i \cdot 0,014)\ Pa$;

– piezoelectric modules: $e_{31} = -1,0 \cdot (1 - i \cdot 8,3 \cdot 10^{-3})\ C/m^2$, $e_{33} = 1,5\ C/m^2$, $e_{15} = 1,13 \cdot (1 - i \cdot 2,1 \cdot 10^{-3})\ C/m^2$;

– the components of dielectric properties matrix: $\kappa_{11}/\kappa_0 = 12,7 \cdot (1 - i \cdot 4,7 \cdot 10^{-3})$, $\kappa_{11}/\kappa_0 = 11,8 \cdot (1 - i \cdot 1,2 \cdot 10^{-3})$, $\kappa_0 = 8,85 \cdot 10^{-12}\ F/m$.

Parameters of the fast Fourier transform: number of points $N = 1000$ on the time interval $T = 0,4 s$.

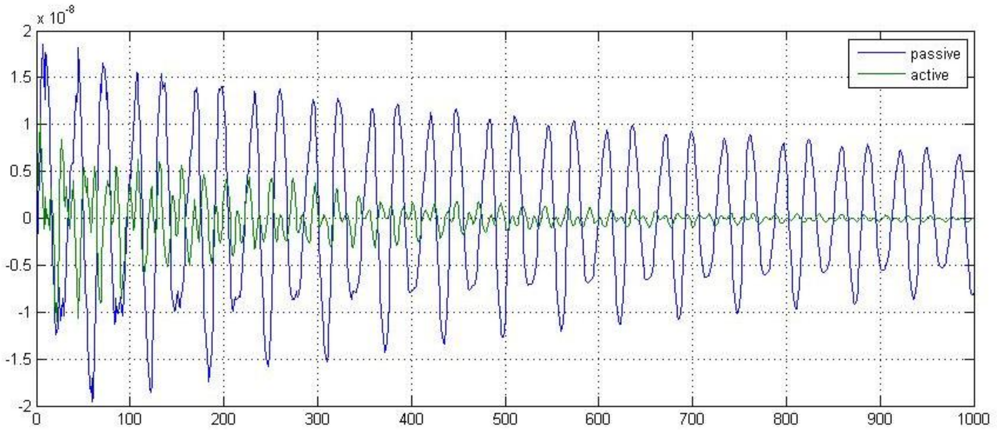
The solution of the problem is considered for two cases:

1) passive damping by RL-shunts. In this case the matrix of dynamic stiffness looks like:

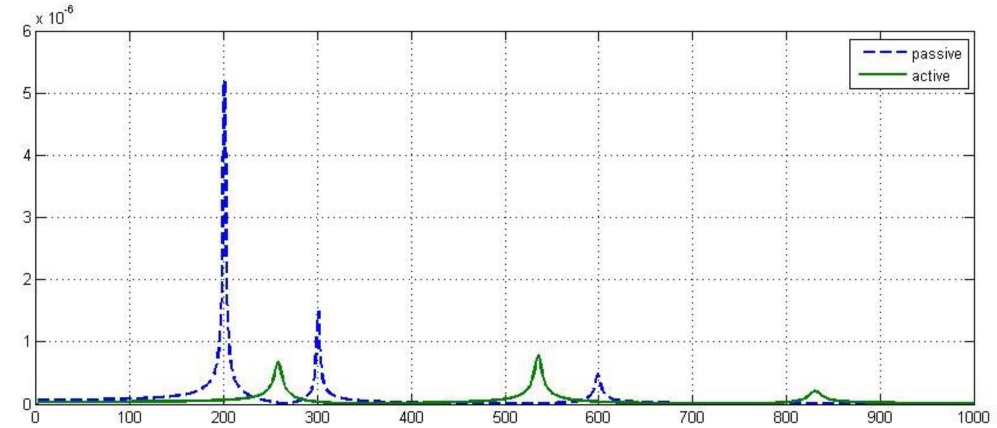
$$\tilde{\mathbf{Z}}(\omega) = -\omega^2 \mathbf{M} + \tilde{\mathbf{K}}_u + \tilde{\mathbf{K}}_{u\varphi}^{s} \left(\tilde{\mathbf{Q}}_s(\omega) + \tilde{\mathbf{K}}_{\varphi}^{s} \right)^{-1} \tilde{\mathbf{K}}_{u\varphi}^{s\,T}. \tag{12}$$

2) active damping of nonstationary vibration in a beam. In this case the matrix of dynamic stiffness (7) is used.

The reaction of the beam to a shock load is given in fig. 2, *a*. The frequency response is given in fig. 2, *b*.



a



b

Fig. 2. Reaction of the beam to a shock load (a); frequency response (b)

Decrement of nonstationary vibration in case of active damping equals $\Delta = 0{,}0203$.

Optimization of the feedback control parameter. As shown in [13], the choice of optimal parameters for a passive shunt can increase the speed of vibration fading in structures under dynamic loads. Let's consider the problem in case of active damping of nonstationary vibration. This optimization problem lies in calculating the project parameter G by the criterion of maximum damping, taking into account the constraints on characteristics of control devices. The given optimization task can be formulated as a generalized nonlinear programming problem [10]:

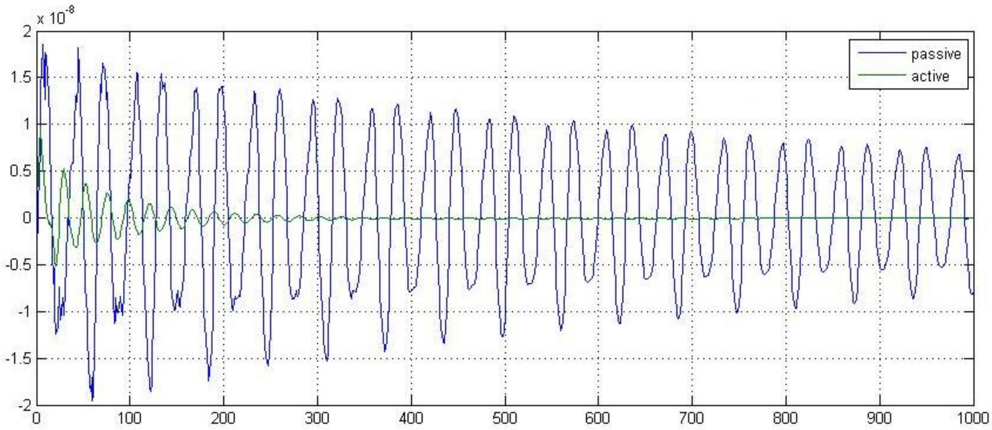
$$\max \Delta(G, u). \tag{13}$$

The constraints for the project parameters

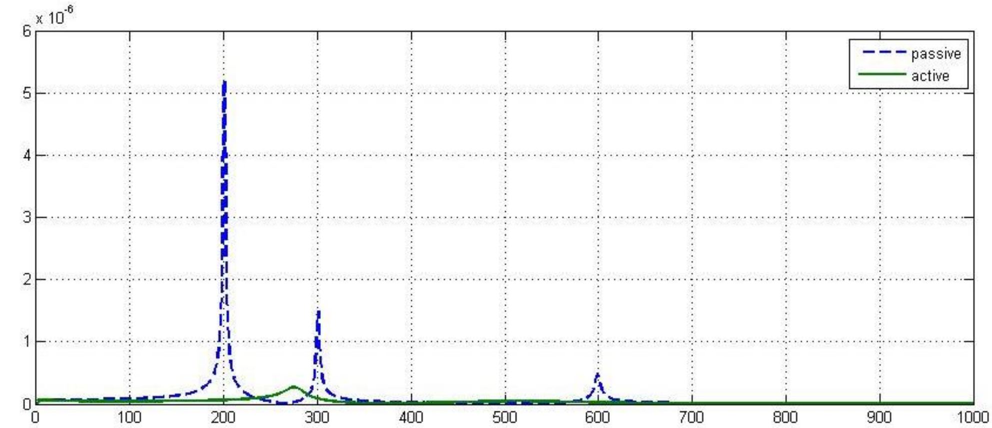
$$G_{\min} \le G \le G_{\max}, \tag{14}$$

are chosen regarding the technical characteristics of devices that implement the selected feedback algorithm.

The optimal goal function value was found in 14 iterations using the *fmincon* program [14]. The reaction of the beam to a shock load and the frequency response for the optimal value G_{opt} are shown in fig. 3.



a



b

Fig. 3. Reaction of the beam to a shock load (a) and frequency response (b) for an optimal value of the amplification coefficient $G_{opt} = 10^{-4}$

The decrement of nonstationary vibration in case of active damping with optimal control parameter G_{opt} equals $\Delta_{opt} = 0{,}4808$.

Conclusions. The results of calculating the nonstationary vibration of a piezoelectric beam with active electroviscoelastic elements lead us to a conclusion that using sensors and actuators, working in opposition, can provide significant decrease of vibration amplitude for a given type of structure. The application of active damping technique allows to change the vibration decrement of a structure by regulating the amplification coefficient G. The efficiency of this technique depends on the accuracy and stability of devices that provide the feedback of sensor and actuator. It is important to note that the optimal value of the control parameter G_{opt} resulted in practically negligible level of resonant beam vibration.

The finite element analysis in Fourier transform space can be applied for studying the behavior of active piezoelectric structures and composites, based on them, under shock loads. The advantages of this technique are the possibility to determine the reactions to external loads and take into account the dependencies between frequency and physical characteristics of a material. The return to the time space is done only at the last step of calculations.

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