
This paper is a previously accepted version of the article.
The final published version is available in IEEE Xplore Digital Library
http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=7146934&isnumber=7146825

© 2015 IEEE. Institute of Electrical and Electronics Engineers.
Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works.

Даний матеріал є версією статті, прийнятої до публікації.
Кінцева опублікована версія статті доступна в електронній бібліотеці IEEE Xplore
http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=7146934&isnumber=7146825

© 2015 IEEE. Інститут інженерів з електротехніки та електроніки.
Особисте використання цього матеріалу дозволяється. Дозвіл від IEEE повинен бути отриманий для всіх інших видів використання, в будь-яких поточних або майбутніх засобах масової інформації, у тому числі передрук / переводження цього матеріалу для реклами або рекламних цілей, створення нових колективних робіт, для перепродажу або розповсюдження по серверам або спискам, або повторне використання будь-яких захищених авторським правом компонентів цієї роботи в інших роботах.
A Subharmonic Stability of Power Factor Correctors with Dual-Loop Control System

Yuriy Denisov and Serhii Stepenko
Department of Industrial Electronics
Chernihiv National University of Technology
Chernihiv, Ukraine
Serhii.Stepenko.UA@ieee.org

Abstract—This work is devoted to investigation of the power factor corrector’s (PFC) dual-loop control systems. Two control systems of PFC with different structures were studied: with main voltage and main current feedbacks. A subharmonic stability analysis has been performed for PFC based on conventional boost converter. However, the expression for input current of the storage inductor can be used for PFC based on zero-current switching quasi-resonant boost converter. The stability condition at the main subharmonic frequency has been formulated. The calculation results of the voltage change rate at the current loop input and factor of pulsations have been presented. The PFC subharmonic stability margin has been assessed and condition for choosing the reference voltage amplitude has been proposed.

Keywords—boost converter; control system; current feedback; dual-loop control; energy efficiency; power factor correction (PFC); subharmonic stability; voltage feedback

I. INTRODUCTION

The studies devoted to the problems of energy efficiency and particularly on the development of power factor correctors (PFC) have increasing relevance worldwide. Thus, in Micro Grids an active rectifier with power factor correction must be used to resolve the problem of non-sinusoidal currents [1]. An energy efficiency analysis of various PFC circuits and its energy performance estimation under different control methods are quite important issues [2], [3].

A power part of the structure of PFC usually includes a two-stroke single-phase diode rectifier and high-frequency boost converter (Fig. 1). Its informational part comprises a control system, current and voltage sensors included in the corresponding feedback channels. Due to parameters inequality of rectifier diodes the basic subharmonic may appear in the output voltage curve, whose frequency is two times lower than the fundamental frequency of the rectified voltage. Since the filter units of PFC will not have a significant effect on its amplitude, under appropriate conditions (balance of amplitudes and balance of phases) the undamped self-oscillations may occur at the basic subharmonic frequency.

Although the most of works are focused on analysis of the input current spectrum [4], [5] and dynamic processes in PFC [6], [7], unfortunately, there is quite limited amount of works devoted to the problems of PFC subharmonic stability [8], [9], [10]. In some articles these issues have not got enough attention [11].

II. CONTROL SYSTEMS DESCRIPTION

The causes of subharmonic oscillations in closed-loop structures of power electronics devices and possible ways to solve such problems are studied in details in [12].

The most of PFC systems as usual are built based on conventional boost converters [1], [3], [5], [9]. From the other hand, the PFC system based on zero-current switching quasi-resonant boost converter is prospective subject for further research due to its high-frequency operation with high performance [4], [6], [13].

The input current spectrum analysis for PFC power stages based on conventional and quasi-resonant boost converters was performed in [4] and the regulation accuracy for dual-loop control systems of PFC was estimated in [13], that affects significantly on PFC overall energy efficiency. However, the issue of subharmonic stability of PFC with dual-loop control system remains omitted.

Moreover, there are currently no works on PFC subharmonic stability analysis with proposed results which could be used for PFC based on conventional boost converter as well as for PFC based on zero-current switching quasi-resonant boost converter. This work aims to address the mentioned challenge.

Fig. 1. The power stage of the power factor corrector
The structures of these PFC control systems are presented in Fig. 2. The blocks and symbols mean the following:

- \( \Delta v \), \( \Delta c \) are voltage and current regulation errors; 
- \( K_v(p) \), \( K_c(p) \) are transfer functions of regulators in voltage and current loops; 
- CtrS is control system; 
- PE is pulse element; 
- \( W_{vL}(p) \), \( W_{cl}(p) \) are voltage and current transfer functions of the load; 
- \( U_{ref} \), \( i_{ref} \) are reference signals of the voltage and current loops; 
- \( U(n) = U_{ref} \sin(\omega g \cdot t \cdot n) \) is the input voltage of boost converter; \( n = 1, 2, 3, \ldots N \); 
- \( \omega g \) is the relative frequency of the grid voltage; 
- \( T_s \) is working period of the power switch SW; 
- \( N \) is the number of switching periods within one period of the rectified voltage fundamental frequency; 
- \( R_L \) is the load resistance; 
- CVC is current-voltage converter; 
- CS, VS are current and voltage sensors; 
- \( i_{in}(n) \) is the input inductor current.

There are two switching intervals within the working period of a boost converter associated with accumulation energy by inductor \( L \) (when switch SW is open) and transfer the energy to the load while it is connected to the mains (when switch SW is closed).

The current and voltage transfer functions of the load for the second switching interval represented by (1) and (2) respectively, where \( \omega_f = \frac{1}{\sqrt{LC_f}}, \xi = \frac{\rho}{2R_L} \), \( \rho = \frac{L}{C_f} \).

\[
W_{cl}(p) = \frac{\omega^2}{\omega^2 + 2\zeta \omega \rho + \rho^2} \\
W_{cl}(p) = \frac{\omega^2}{\omega^2 + 2\zeta \omega \rho + \rho^2}.
\]

In [6] the values of discrete transfer functions for open current \( W_{oc}^{*} (z,1) \) and voltage \( W_{ov}^{*} (z,1) \) loops, as well as for closed current \( W_{cc}^{*} (z,1) \) and voltage \( W_{cv}^{*} (z,1) \) loops for dual-loop control system of PFC were obtained. Taking into account their values, the current transfer functions for closed-loop PFC control systems at the switching moment of pulse element \( \varepsilon = 1 \) were obtained: for the main current feedback (3) and for the main voltage feedback (4).

\[
W_{oc}^{*} (z,1) = \frac{W_{oc}^{*} (z,1) \cdot W_{cc}^{*} (z,1)}{1 + W_{oc}^{*} (z,1) \cdot W_{ov}^{*} (z,1)}, \\
W_{cv}^{*} (z,1) = \frac{W_{cc}^{*} (z,1)}{1 + W_{ov}^{*} (z,1) \cdot W_{cc}^{*} (z,1)}.
\]

After appropriate transformations using [6] one can write the complete expressions for PFC with main current loop (5) and main voltage loop (6):

\[
W_{cc}^{*} (z,1) = K_{oc} \left( B_{o1}^{*} z^2 - B_{o2}^{*} z + B_{o3}^{*} \right) / \left( z^3 - 2 e^{-z} \cos \alpha \omega \cdot z^2 + e^{-2z} \right) + K_{oc} \left( B_{o1}^{*} z^2 - B_{o2}^{*} z + B_{o3}^{*} \right), \\
W_{cv}^{*} (z,1) = K_{oc} \left( z^3 - 2 e^{-z} \cos \alpha \omega \cdot z^2 + e^{-2z} \right) / \left( z^3 - 2 e^{-z} \cos \alpha \omega \cdot z^2 + e^{-2z} \right) + K_{oc} \left( z^3 - 2 e^{-z} \cos \alpha \omega \cdot z^2 + e^{-2z} \right).
\]
where \( \bar{a} = \hat{z} \omega_f \); \( K_0 = U(n)K_{V0}K_{C0}; K_{OC} = \frac{U(n)K_{OC}K_{C0}}{R_L} \).

The control system gain at a relative switching period of boost converter equal to unity \( K_{OC} = F(n)/U_{SM} \), where \( U_{SM} \) is a reference sawtooth voltage amplitude of the control system, \( F(n) \) is a factor of pulsations. Expressions (5) and (6) were obtained under condition \( K_{C}(p) = K_{V}(p) = 1 \) and the coefficients \( B^C, B^C, B^C, B^C, B^C, B^C \) were obtained in [14].

Based on (5), (6) one can write the characteristic equations of the closed-loop control system of PFC for main voltage feedback (7) and main current feedback (8):

\[
\begin{align*}
\bar{z}^3 - z^2 (2 e^{-\bar{z}} \cos \bar{\omega}_0 - K_{V0} B^R_0) + z \left( e^{-\bar{z}} - K_{V0} B^R_0 \right) + K_{OC} B^R_0 (\bar{z} - B^C_{21} z + B^C_1) &= 0, \\
\bar{z}^3 - z^2 (2 e^{-\bar{z}} \cos \bar{\omega}_0 - K_{V0} B^R_0) + z \left( e^{-\bar{z}} - K_{V0} B^R_0 \right) + K_{OC} B^R_0 (\bar{z} - B^C_{21} z + B^C_1) &= 0, \\
\end{align*}
\]

(7)

A high-frequency power switch in PFC is working with relative frequency \( \bar{\omega}_0 = 2 \pi \). It is known [14] that the discrete systems are characterized by the periodicity of the frequency response.

This property is retained for the complex transfer function, and hence for the frequency response of the pulse system, i.e.

\[
W^*(j\bar{\omega}_0 + 2\pi n, \bar{e}) = W^*(j\bar{\omega}_0, \bar{e}) = W^*(j\bar{\omega}_0, \bar{e}),
\]

where the relative frequency of the rectified voltage pulsations is \( \bar{\omega}_0 = \omega_0 T_s \) and \( T_s = N T_s \).

III. SUBHARMONIC STABILITY ANALYSIS

The abovementioned feature of the discrete systems allows investigating the PFC stability at the main subharmonic relative frequency \( \bar{\omega}_0 = \pi \), which is twice times lower than the relative frequency of the rectified voltage \( \bar{\omega}_0 = \pi \), using the frequency characteristics obtained from equations (7), (8), in which

\[
z = e^{-\omega L_0} = \cos \omega_0 + j \sin \omega_0.
\]

For \( \bar{\omega}_0 = \pi \) using (7), (8), one can write the values of PFC frequency characteristics at the main subharmonic frequency, which have the same values for the structures in question:

\[
K_0 \left( \sum B^C + \frac{K_{CS}}{R_s} \sum B^C \right) = M,
\]

(10)

where

\[
K_0 = U(n)F(n)/U_{SM}, \quad \sum B^C = B^C_0 + B^C_0 + B^C_0, \quad \sum B^C = B^C_0 + B^C_0 + B^C_0, \quad M = 1 + 2 e^{-\bar{z}} \cos \bar{\omega}_0 + e^{2\bar{z}}.
\]

Thus, from (10) we obtain the PFC stability condition with the main voltage loop and main current loop at the main subharmonic frequency

\[
F(n) \leq \frac{M}{U(n) \left( \sum B^C + \frac{K_{CS}}{R_s} \sum B^C \right)}.
\]

(11)

Obviously, the PFC has infinite stability when \( U(n) \to 0 \) and minimal stability when \( U(n) \) reaches maximal value. In this case a significant effect on the region of subharmonic stability will have a factor of pulsations \( F(n) \). It largely depends on the rate of change of inductor current \( i_n(n) \), which under small value of inductance and high input voltage may be significant.

Let us find the value of factor of pulsations, using the rate of change of the inductor current \( i_n(n) \) at the ON time point of pulse element (\( \epsilon = 1 \)).

Under the analysis of processes in parallel pulse converter [13], the expression for steady state current \( i_{n}^t(\bar{t}) \) was obtained (12) within the time interval \( n + \gamma \leq \bar{t} \leq n+1 \), when the PFC load is connected to supply voltage.

\[
i_{n}^t(\bar{t}) = \frac{L_0^2}{R_s \bar{\omega}_0} e^{-\bar{\omega}_0(t-n-\gamma)} (\sin \bar{\omega}_0(t-n-\gamma) + \omega_0 C_s R_s) i_n(n) d +
\]

\[
+ A(n) - \frac{C_s R_s \omega^2_0}{4 \bar{\omega}_0} e^{-\bar{\omega}_0 (t-n-\gamma)} \sin \bar{\omega}_0(t-n-\gamma) i_n(n) + A(n),
\]

(12)

Putting \( \bar{t} = n + \epsilon \) and \( \epsilon = \frac{\Delta t}{T_s} \), one can write:

\[
i_{n}^t(\epsilon) = \frac{L_0^2}{R_s \bar{\omega}_0} e^{-\bar{\omega}_0(\epsilon-(\gamma-\bar{t}))} (\sin \bar{\omega}_0(\epsilon-(\gamma-\bar{t})) + \omega_0 C_s R_s) i_n(n) d +
\]

\[
+ A(n) - \frac{C_s R_s \omega^2_0}{4 \bar{\omega}_0} e^{-\bar{\omega}_0 (\epsilon-(\gamma-\bar{t}))} \sin \bar{\omega}_0(\epsilon-(\gamma-\bar{t})) i_n(n) + A(n),
\]

(13)

where \( i_n(n) = M(n) (1 - F(\gamma))^{-1} \) is an inductor steady state current at \( \bar{t} = n \), \( M(n) = A(n) + \frac{L_0^2}{R_s \bar{\omega}_0} \sin \bar{\omega}_0(1-\gamma) - \frac{C_s R_s \omega^2_0}{4 \bar{\omega}_0} \sin \bar{\omega}_0(1-\gamma), \) \( A(n) = \frac{U_s}{\omega_0 L} \cos \omega_0 n - \cos \omega_0 (n + \gamma) \),

\[
F(\gamma) = e^{-\bar{\omega}_0 (\epsilon-(\gamma-\bar{t}))} \left( d + \frac{L_0^2}{R_s \bar{\omega}_0} \sin \bar{\omega}_0(1-\gamma) d - \frac{C_s R_s \omega^2_0}{4 \bar{\omega}_0} \sin \bar{\omega}_0(1-\gamma) \right).
\]

For the PFC based on a zero-current switching quasi-resonant boost converter, discussed in [4], [6], [13], coefficient \( d = 1 - \frac{R_s}{\omega_0 L \sin \bar{\omega}_0 \epsilon} \), where \( \bar{\omega}_0 = \omega_0 T_s \) is the relative switching frequency and \( \omega_0 = \frac{1}{\sqrt{L_s C_s}} \) is the frequency of the resonant circuit.

For the PFC based on a conventional boost converter (Fig. 1), e.g. investigated in [2], [3], [5], [7], where the resonant circuit is absent, coefficient \( d = 1 \).
Expression (13) can be written as follows
\[
 i_2(\varepsilon) = a_1 e^{\tau(e-e)} \sin \omega_0(\varepsilon - \varepsilon) + a_2 e^{\tau(e-e)} - \\
 - a_3 e^{\tau(e-e)} \sin \omega_0(\varepsilon - \varepsilon), \quad \gamma \leq \varepsilon \leq 1,
\] (14)
where the coefficients \(a_i\) are as follows:
\[
a_1 = \left( i_1(n) + A(n) \right) \cdot \frac{L_i \omega^2}{R_i \omega},
\]
a_2 = i_1(n) + A(n),
\[
a_3 = \left( A(n) + i_1(n) \right) \cdot C_i \cdot R_i \cdot \omega^2.
\]

A signal from the current sensor \(U_{CS}(\varepsilon) = R_{CS}i_2(\varepsilon)\).

In view of (14) one can write the rate of change of the current loop signal in the control system for \(\varepsilon = 1\):
\[
\frac{dU_{CS}(\varepsilon)}{d\varepsilon} = R_{CS}e^{\tau(e-e)} \left( a_1 \left( \bar{\omega}_0 \sin \bar{\omega}_0(1 - \gamma) - \bar{\omega} \sin \bar{\omega}_0(1 - \gamma) \right) - \\
- a_2 \bar{\omega}_0 \sin \bar{\omega}_0(1 - \gamma) - \bar{\omega} \sin \bar{\omega}_0(1 - \gamma) \right),
\] (15)
A factor of pulsations as known [14] is as follows:
\[
F = \frac{1}{1 - \frac{dU_{CS}(\varepsilon)}{d\varepsilon} \frac{dU_{REF} / d\varepsilon}{dU_{CS}(\varepsilon) / d\varepsilon}}.
\] (16)

Let us calculate the rate of change of the voltage applied to the current loop input in PFC control system and the factor of pulsations for the following conditions:
\[L = 0.825 \cdot 10^{-3} \text{H}, \quad \bar{\omega}_0 = 1.1 \cdot 10^{-2}, \quad \bar{\omega} \equiv \bar{\omega}_0, \quad \rho = 0.91 \text{Ohm},\]
\[\bar{\xi} = 4.54 \cdot 10^{-2}, \quad \bar{a} = 0.5 \cdot 10^{-3}, \quad R_{CS} = 1 \text{Ohm}.
\]

The results of calculation are presented in Table I.

**TABLE I.** **VOLTAGE RATE OF CHANGE AND FACTOR OF PULSATIONS**

<table>
<thead>
<tr>
<th>n</th>
<th>U(n), V</th>
<th>(\Sigma B^p)</th>
<th>(\Sigma B^c)</th>
<th>SM</th>
<th>F(n)/U_SM</th>
<th>N(M)/U(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>125</td>
<td>8.19</td>
<td>4.512</td>
<td>0.463</td>
<td>0.1456</td>
<td>0.39 \cdot 10^{-2}</td>
<td></td>
</tr>
<tr>
<td>250</td>
<td>8.19</td>
<td>4.512</td>
<td>0.463</td>
<td>0.1326</td>
<td>0.21 \cdot 10^{-2}</td>
<td></td>
</tr>
<tr>
<td>375</td>
<td>8.19</td>
<td>4.512</td>
<td>0.463</td>
<td>0.1216</td>
<td>0.16 \cdot 10^{-2}</td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>8.19</td>
<td>4.512</td>
<td>0.463</td>
<td>0.1166</td>
<td>0.14 \cdot 10^{-2}</td>
<td></td>
</tr>
</tbody>
</table>

As could be seen from Table I the factor of pulsations depends on the magnitude of the input voltage, however, this relationship is not abrupt.

The factor of pulsations has a stabilizing effect on the system, since its value is less than unity. However, condition (11) in PFC, wherein the gain depends on the input voltage, is performed only for low voltage magnitudes.

It is obvious that in such PFC the stability margin on the main subharmonic at \(U_{SM} = 5V\) is minimal. It can be seen from Table II, where
\[
N(M) = \frac{M}{\sum B^p + R_{CS} \sum B^c}.
\] (17)

To eliminate the dependence of the gain on the input voltage in the PFC with the main voltage feedback (e.g. IC UC3852) the special blocks are envisaged (rectified voltage squarer, voltage loop error divider, reference current multiplier). The output of the multiplier is connected to the input of the current loop.

In this case the PFC subharmonic stability condition is as follows:
\[
\frac{F(n)}{U_{SM}} \leq N(M).
\] (18)

As could be seen from Table II, this condition at \(U_{SM} = 5V\) is performed for the entire range of input voltage. The selection of the reference sawtooth voltage amplitude of the control system, which will ensure the necessary subharmonic stability margin, could be performed based on this condition.

In the performed analysis assumed that the transfer function of the current controller \(K_C(p) = 1\). In the case of IC UC3852 there is a PI controller in this circuit, the output signal of which will affect the rate of change of the control signal. However, when you consider the factor of pulsations is determined in the mode of infinitely small variations of the control signal near the stationary state, then in the presence of proportional component of the signal at the output of the PI controller, the rate of its change will not be significantly different from the rate of change of the current \(i_1(\varepsilon)\), (15). Therefore, the stability condition (18) can be also extended to such PFC as based on IC UC3852.

In general case the factor of pulsations should be calculated taking into account the derivative of the controller output signal, that is its response for the input inductor current \(i_L(\varepsilon)\).

As for the PFC with the main current feedback, the performed analysis showed that its subharmonic stability margin does not differ from the PFC with the main voltage feedback. Conditions (11) and (18) are valid for both PFC’s characteristic equations (7) and (8).
IV. CONCLUSIONS

The power factor correctors with dual-loop control system were investigated in this work. Those power factor correctors, in which the open structure’s gain depends on the input voltage, are stable at the main subharmonic frequency only under low input voltage values.

The power factor correctors with the main current feedback and main voltage feedback, in which the gain of the open structure is independent from the input voltage, have the same stability margin at the main subharmonic. Its value is determined by the choice of the reference voltage amplitude in the control system.

REFERENCES


