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Y.Zharii, candidate of technical sciences
H.Veremei, senior lecturer
(Chernihiv National University Of Technology, Ukraine, Chernihiv)

THE GEOMETRY MATHEMATICAL MODELLING OF THE OVERHAULED VALVE-SEAT SURFACES IN THE VALVE TIMING GEAR

The analytical methods within geometry modelling of the overhauled valve-seat surfaces in the valve timing gear have been considered. The mathematical model to analyze the state of the working valve-seat surfaces under the wear for the purpose to form their parameters of quality within overhaul's process has been offered.

Generalized 3D - model of the forming process for the valve seat worn surfaces of the valve timing gear in the overhauled repair requires the development of a mathematical model describing the geometry of these surfaces.

The general processing scheme of a valve seat under the wear with description of the constructive and geometrical parameters of the metal-working equipment, affects the quality of the overhaul's process, was submitted in the articles [2, 5].

To describe the surface's state of a worn valve seat [7, 8] the usage of spatial equation in explicit form $z = f(x, y)$ is proposed. Wherein the analytical model of the surfaces is constructed as a generalized polynomial

$$Q(x, y) = \sum_{k=1}^{n} c_k \cdot \varphi_k(x, y), \quad (1)$$

where $c_1, c_2, \dots c_n$ - the coefficients to be determined; $\varphi_1(x, y), \dots \varphi_n(x, y)$ - system of basic functions of a certain functional space must meet the necessary requirements of continuity and smoothness.

To adjust the more precise model will use the interpolation scheme when the function $z = f(x, y)$ and the polynomial $Q(x, y)$ coincide on a given system of points in the S – area (limited with inner and outer radiuses of the valve seat)

$$Q(x_i, y_i) = f(x_i, y_i) = z_i, \ i = 1, \dots n. \quad (2)$$

Unknown model parameters $c_1, c_2, \dots c_n$ are obtained from a system of linear equations [1]:

$$z_i = \sum_{k=1}^{n} c_k \cdot \varphi_k(x_i, y_i), \ i = 1, \dots n, \quad (3)$$

where (x_i, y_i, z_i) - the starting points of the working surfaces of the area S. System (3) has a unique solution if its determinant is not zero:

$$\Delta = \begin{vmatrix} \varphi_1(x_1,y_1) & \varphi_2(x_1,y_1) & \dots & \varphi_n(x_1,y_1) \\ \varphi_1(x_2,y_2) & \varphi_2(x_2,y_2) & \dots & \varphi_n(x_2,y_2) \\ \dots & \dots & \dots & \dots \\ \varphi_1(x_n,y_n) & \varphi_2(x_n,y_n) & \dots & \varphi_n(x_n,y_n) \end{vmatrix} \neq 0. \quad (4)$$

To develop maximally-considered working surfaces model the system of basis functions must be chosen by the way which is able to describe the exact process caused the deterioration of the valve seat. Therefore, to describe irregular surfaces and ensure the fulfillment of condition (4), taking into account the topography of the working part [7, 8] as the basis functions - quadric is proposed to use:

$$\varphi_k(x,y) = \sqrt{(x-x_k)^2 + (y-y_k)^2 + A_\varphi}\ , \quad (5)$$

where A_φ - the parameter that affects the curvature of the modeled surface and can be specified as a experimentally substantiated constant ($A_\varphi \geq 0$). Computation of the determinant (4) and the solution of linear equations (3) are carried out using matrix functions of MathCAD [4].

Spatial model (1) can be used for further three-dimensional analysis of the valve seat worn surfaces and settings before surface's overhaul. To this purpose the extremes (maxima and minima) of the original surface of each of the chamfers *(A, B, C)* through the points $(x,y) \in S$ should be researched:

$$Q_{\min}^{A,B,C} = \min[Q(x_i,y_i)],\ Q_{\max}^{A,B,C} = \max[Q(x_i,y_i)],\ i = 1,\dots n. \quad (6)$$

At the found minimum points (x_j, y_j) $(j = 1,\dots k)$ for each chamfer is possible to calculate the value $Z_{\min}^{A,B,C}$ of the processed surfaces for the base from the general equation of a right circular cone

$$Z_{\min}^{A,B,C} = \min[Z(x_i,y_i)]\ ,\ Z(x,y) = p \cdot \sqrt{x^2 + y^2} \quad (7)$$

where *p* - parameter defined for each of the facets [10], based on the standards of forming parameters of the finished valve seat:

$$p_A = \frac{(R_1 - R_0)\cdot tg\alpha_1}{R_1},\ \ p_B = \frac{(R_2 - R_1)\cdot tg\alpha_2}{R_1},\ \ p_C = \frac{(R_3 - R_2)\cdot tg\alpha_3}{R_2}. \quad (8)$$

The decision on the feasibility and advisability of the parts can be taken upon detection of deeper than $Z_{\min}^{A,B,C}$ - the original surface depressions on at least one of the chamfers of the valve seat *(A, B, C)*, that is when

$$Q_{\min}^{A,B,C} < Z_{\min}^{A,B,C}. \quad (9)$$

If worn seat is suitable for processing, the maximum deviation of the peaks and valleys across the surface to be overhauled can be determined (1):

$$\Delta_{\max}^{Q} = \max_{A,B,C}\left| Q_{\max}^{A,B,C} - Q_{\min}^{A,B,C} \right|, \quad (10)$$

that allows to pick the best overall allowance for processing,

$$Z_{общ} = \Delta^{Q}_{\max} . \quad (11)$$

Three-dimensional analysis allows highly-precise degree to control of roughness parameters as at any point of the predetermined area S can be processed to determine the function of the deviation of the surface (1) for the base (7):

$$r(x, y) = Q(x, y) - Z(x, y) . \quad (12)$$

Then the average R_a and RMS R_q asperity in accordance with the standards of ISO-4287 [3] can be calculated analytically:

$$R_a = \frac{1}{S}\iint_S |r(x, y)| dxdy , \ R_q = \sqrt{\frac{1}{S}\iint_S r^2(x, y) dxdy} . \quad (13)$$

Dimensional analysis involves the use of cross sections obtained by the intersection of 3D-model of the overhauled surface with define planes. In the general case, the slicing plane might be taken a plane of a general position, then the line of intersection with surface its can be defined (1) by the system of equations

$$\begin{cases} A_1 x + B_1 y + C_1 z + D_1 = 0 \\ z = Q(x, y). \end{cases} \quad (14)$$

A solution of (14) can be obtained by any of the numerical [1] techniques (iterations of Newton's method, etc.), however, more efficient, from the standpoint of the computer implementation, the transition from a problem to a differential algebraic [6]. For this purpose the system can lead to a nonlinear equation

$$F(x, y) = A_1 x + B_1 y + C_1 Q(x, y) + D_1 = 0, \quad (15)$$

where the function of line of intersection - *y (x)* is contained in an implicit form, and its derivative has the form

$$\frac{dy}{dx} = -\frac{F'_x(x, y)}{F'_y(x, y)} = \Phi(x, y). \quad (16)$$

Setting the initial approximation for the required line in implicit form ($y_0 = y(x_0)$, $z_0 = z(x_0)$) and using the numerical solution of (15) Runge-Kutta method [4], the equation of the curve in space

$$\begin{cases} y = y(x) \\ z = z(x). \end{cases} \quad (17)$$

If to apply to the system (14) the intersecting vertical planes passing through the axis Oz (when $C_1 = D_1 = 0$) it is possible to construct a family of plane curves (17) of the form $z = z(x)$ in the coordinates profile and to use them to control the taper cutting zone. Having built in the *j-th* processing step (Fig. 2, b) [10] on the points of the curve $z_i = z(x_i)$ *(i = 1, ... n)* linear regression [4] for each of the chamfers (*A, B, C*) the straight-line equation can be obtained:

$$z^{A,B,C}(x) = x \cdot tg\alpha^{A,B,C} + b^{A,B,C} , \quad (18)$$

where $tg\alpha^{A,B,C}$ - the angular coefficient of the line.

Then, through the definition of error built in tilt angles and base line might be evaluated by deviations from the cutting area for each of the taper seat chamfer on the *j-th* processing step:

$$\Delta\alpha^{A,B,C} = arctq(tg\alpha^{A,B,C}) - \alpha_{1,2,3}\,. \qquad (19)$$

To analyze the roundness of the chamfers in the first equation (14) the horizontal plane ($A_1 = B_1 = 0$, $z = const$) is used allowing to build a profile of a family of plane coordinates curves of the form $y = y(x)$. Each such plane curve can be investigated by calculating the roundness on deviations from the basic points of the circle (for example, through the construction of non-linear regression [4]). However, if in the course of processing the estimated standard error almost unchanged reasons set using this method, is more complicated. It is therefore proposed to use a piecemeal approach - through the construction of auxiliary circles (internal and external), circumscribed about triangles.

For this (by selection or using functions [4]) three points are selected which most closely spaced from the origin $O(x_0, y_0)$ (the center of the base) and three ones - from the most distant point of $O(x_0, y_0)$ (Fig. 3, a) [10].

Through nearby points $A_m(x_1^m, y_1^m)$, $B_m(x_2^m, y_2^m)$, $C_m(x_3^m, y_3^m)$ the inner circumscribed circle is constructed, through the outermost point - $A_M(x_1^M, y_1^M)$, $B_M(x_2^M, y_2^M)$, $C_M(x_3^M, y_3^M)$ the outer inscribed circle.

To calculate the inner R_M and outer R_m radiuses of the circumscribed circles the sinus and cosines theorem for triangles composed of selected points [1]. For a triangle (Fig. 3,b) [10] formulas are:

$$R = \frac{c}{\sqrt{1 - \cos^2\gamma}}, \qquad (20)$$

where $a = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$; $b = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$;

$c = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$; $\cos\gamma = \dfrac{a^2 + b^2 - c^2}{2ab}$.

Then the calculation of the radial spread

$$\Delta R = R_M - R_m \qquad (21)$$

allows to estimate the degree of similarity of the curve and a circle. At the initial stage of the processing of such variation in the radiuses can be significant due to the topographical features of the surface finish. At the final stage R_M and R_m the must match the allowance of the chamfer radius.

Center of the circumscribed circle around the triangle is the point of intersection of the middle perpendicular to its sides (Fig. 3, b) [10]. To define the coordinates of the center a system of linear equations of the form can be used:

$$\begin{cases}(x_1 - x_2)(x - \frac{x_1 + x_2}{2}) + (y_1 - y_2)(y - \frac{y_1 + y_2}{2}) = 0 \\ (x_2 - x_3)(x - \frac{x_2 + x_3}{2}) + (y_2 - y_3)(y - \frac{y_2 + y_3}{2}) = 0\end{cases}, \qquad (22)$$

The solution of equation system (22) will give the possibility to calculate the coordinates of the centers of $O_M(x_M, y_M)$ - external and $O_m(x_m, y_m)$ - internal circles. Then the calculation of the maximum deviation of the obtained centers O_M or O_m from the basic one $O(x_0, y_0)$ will allow to assess the degree of coaxiality of the coordinate systems in the processing of the valve seat.

Conclusions

Within considering the analytical methods for geometry modelling of the overhauled valve-seat surfaces in the valve timing gear have been the mathematical model to analyze the state of the working valve-seat surfaces under the wear for the purpose to form their parameters of quality within overhaul's process.

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