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INFLUENCE OF THERMAL HETEROGENEITY ON DEFORMATION OF THIN CIRCULAR CYLINDRICAL SHELL

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Summary. *Thermal deformation of the long cylindrical shell which is taking place in an external temperature field is considered. It is supposed that the material of the shell is heterogeneous, which results in dependence of factor of thermal expansion on angular coordinate θ. Differential equations of the shell thermoelastic deformation are written for the plane case. The solution is constructed as Fourier series on district coordinate. In the case of change of the thermal expansion coefficient in the range of rotational position, analytical expressions are found for the radial and circumferential displacements of the surface points. Numerical studies that allow to determine the nature and the zone of heterogeneity are conducted.*

K***ey words:*** *thermal deformation, non-linear deformation, cylindrical shell, Fourier series.*

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Problem setting. Thin-walled shell type structural elements are used in civil and industrial construction, in engineering, in rocket – space technology and in other industries. The tasks of thermal conductivity and deformation of the thin shells, which are subjected to temperatures, constitute an important class of problems and is the subject of systematic research in the mechanics of deformable solid body. This is not only because of the interest to theoretical tasks of the thermoelasticity of bodies, which have the features of the geometric structure, but because of the widespread use of the shells in modern designs in terms of uneven heating. Deformation and stress, resulting from heat, significantly affect the bearing capacity of structures, and in many cases the calculation of the shells on thermal effect is crucial.

Analysis of the known research results. A large number of publications on the research of stress state of the shell with thermal effects is a response to the urgent problems of modern industries. With careful analysis of scientific works of domestic and foreign authors we can conclude that intensive development of that theory and development of methods for calculating the thermal and stress states still continue, which are characterized by the simplicity of implementation. Such methods can be developed through reasonable simplification of the initial equations and setting objectives [1], [2]. Setting and characteristic methods of solutions that are based on the use of assumptions about the relative role of a factor, are presented in the works [3], [5 – 7], and others. It should be noted that special attention is paid to the problems of thermomechanics to the way of the thermal loading problem and its modeling in solving the specific problems. In this area, a number of researches have been conducted which included different forms of modeling of the thermal loading: setting values of temperature and the density of heat flow at the border, concentrated heat sources, homogeneous heat flow at infinity, and others. The works of V.R. Karnauhov [4], Y.S. Pidstryhach [3], V.V. Meleshko, A.F. Ulitko [4] are dedicated to the review of the main models and methods of thermoelasticity. It should be mentioned that the solution of dynamic problems of thermal stresses is associated with significant mathematical difficulties. Therefore, in a closed form only some one-dimensional problems have been solved mainly with constant coefficients of linear thermal expansion.

The purpose of the work. To study thermal deformation of the long cylindrical shell which is taking place in an external temperature field provided that the coefficient of linear thermal expansion depends on the angular coordinates. To carry out numerical calculations and

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generalize their results in order to identify the impact of heterogeneity on moving points of the median surface of the shell.

Formulation of the problem. In this paper, to study the thermal deformation of the shell which is taking place in an external temperature field, the original three-dimensional equations are replaced by two-dimensional equations. For the plane case differential equations of thermoelastic deformation of the long cylindrical shell taking into account the thermal deformation and the dependence of the coefficient of linear thermal expansion of angular coordinate θ is a simple generalization of the known equations of S.P. Tymoshenko. It should be noted that in the theory of thermoelasticity the most common method of constructing of two-dimensional problems is a method that is based on the representation of solutions in the form of Fourier series on the degrees of normal coordinate. If during the construction of three-dimensional problem to a two-dimensional one no assumptions are done in regard to temperature distribution on the shell thickness, then a three-dimensional problem is equivalent to the problem of infinite dimensional system of two-dimensional equations in regard to the unknown functions which are the coefficients of expansion. These equations can be obtained in case you add value of thermal deformation into the equation of that theory, that is to put

$$\begin{aligned} \varepsilon_Z &= \frac{1}{Eh}\left(N_x - \nu N_\theta\right) + \alpha_T(\theta)\left(T - T_0\right) \\ \varepsilon_\theta &= \frac{1}{Eh}\left(N_\theta - \nu N_x\right) + \alpha_T(\theta)(T - T_0) \end{aligned} \tag{1}$$

It should be noted that other values are not changed. In the formulas (1) by means of $\alpha_T(\theta)$ the thermal expansion coefficient is indicated, and $T - T_0 = T^*$ – is the heating of the shell. As the result we come to a system of differential equations for the radial and circumferential displacements of the surface points of the middle surface shell (plane problem)

$$\begin{aligned} &\frac{1}{a}\frac{\partial^2 u}{\partial\theta^2} - \frac{1}{a}\frac{\partial\omega}{\partial\theta} + \frac{h^2}{12a^3}\left(\frac{\partial^3\omega}{\partial\theta^3} + \frac{\partial^2 u}{\partial\theta^2}\right) = (1+\nu)\frac{\partial\alpha_T(\theta)}{\partial\theta}\left(T - T_0\right) \\ &\frac{1}{a}\frac{\partial u}{\partial\theta} - \frac{\omega}{a} - \frac{h^2}{12a^3}\left(\frac{\partial^4\omega}{\partial\theta^4} + \frac{\partial^3 u}{\partial\theta^3}\right) = -\frac{qa\left(1-\nu^2\right)}{Eh} + (1+\nu)\alpha_T(\theta)\left(T - T_0\right) \end{aligned} \tag{2}$$

In the written equations $\omega(\theta)$ and $u(\theta)$ refer to radial and circumferential displacement of the surface points of the middle surface shell. Here are the following markings:

h – thickness of the shell, a – the radius of the middle surface, E – Young's modulus, ν – Poisson's ratio, q is used to mark normal mechanical loading on the shell.

As a simple example, let’s consider the case when the coefficient of linear thermal expansion has a simple dependence on the angular coordinate

$$\alpha_T = \alpha_T^0\left(1 + \varepsilon\cos 2\theta\right), \tag{3}$$

where ε – is small compared with unity of dimensionless number. Assuming that external mechanical loading is absent, equations (2) will look like

$$\frac{d^2u}{d\theta^2}-\frac{d\omega}{d\theta}+\frac{h^2}{12a^2}\left(\frac{d^3\omega}{d\theta^3}+\frac{d^2u}{d\theta^2}\right)=(1+\nu)\frac{d\alpha_T(\theta)}{d\theta}T^*a$$
$$\frac{du}{d\theta}-\omega-\frac{h^2}{12a^2}\left(\frac{d^4\omega}{d\theta^4}+\frac{d^3u}{d\theta^3}\right)=(1+\nu)\alpha_T(\theta)T^*a \qquad (4)$$

Assuming that the heating of the shell is made by constant temperature field $T-T_0=T^*=const$, the solution of the equation (4) will look like (5)

$$\omega(\theta)=C+B\cos 2\theta$$
$$u(\theta)=A\sin 2\theta \qquad (5)$$

Constants A, B i C are determined after substitution (5) into equation of elastic thermal balance (4). Then we have

$$A=\frac{2}{3}(1+\nu)aT^*\varepsilon\alpha_T^0$$
$$B=\frac{1}{3}(1+\nu)aT^*\varepsilon\alpha_T^0 \qquad (6)$$
$$C=-(1+\nu)aT^*\varepsilon\alpha_T^0$$

From submitted solution follows that the presence of heterogeneity leads to circular displacements of the middle surface, the law of change is determined by the derivative of the heterogeneity of thermal expansion. At the same time radial displacements are determined by the heat distortion for constant coefficient of thermal expansion.

The most interesting case appears when the coefficient of thermal expansion changes only in a certain range of angular coordinate θ

$$\alpha_T^0=\begin{cases}\alpha_T^0, & \theta_0\le\theta\le\pi,\\ \alpha_T^o+\varepsilon(\cos\theta-\cos\theta_0)^2, & |\theta|\langle\theta_0, \quad \varepsilon\rangle 0\end{cases} \qquad (7)$$

In this case, a complete solution for moving is given in a Fourier series, expanding previously $\alpha_T(\theta)$ also in a Fourier series

$$\alpha_T(\theta)=\alpha_T^0+\varepsilon\left(\frac{\theta_0}{\pi}\right)\left(\frac{1}{2}-\frac{3}{4}\frac{\sin 2\theta}{\theta_0}+\cos^2\theta_0\right)+2\varepsilon\left(\frac{\theta_0}{\pi}\right)\left(\frac{3}{4}\frac{\sin\theta_0}{\theta_0}+\frac{1}{12}\frac{\sin 3\theta_0}{\theta_0}-\cos\theta_0\right)\cos\theta+$$
$$+2\varepsilon\left(\frac{\theta_0}{\pi}\right)\left(\frac{1}{4}-\frac{1}{4}\frac{\sin 2\theta_0}{\theta_0}+\frac{1}{16}\frac{\sin 4\theta_0}{\theta_0}-\frac{\cos\theta_0\sin 3\theta_0}{3\theta_0}+\frac{\cos^2\theta_0\sin 2\theta_0}{2\theta_0}\right)\cos 2\theta+ \qquad (8)$$
$$+\frac{2\varepsilon}{\pi}\sum_{n=1}^{\infty}F_n\cos n\theta,$$

$$F_n=\frac{2\varepsilon}{\pi}\left(\frac{\sin n\theta_0}{n}\left(\frac{1}{2}+\cos^2\theta_0\right)+\frac{\sin(2+n)\theta_0}{4(2+n)}+\frac{\sin(2-n)\theta_0}{4(n-2)}-\cos\theta_0\left(\frac{\sin(1+n)\theta_0}{(1+n)}+\frac{\sin(1-n)\theta_0}{(1-n)}\right)\right)$$

It should be noted that the solution of equations (4) in this case is in the form:

$$\omega(\theta)=\sum_{n=0}^{\infty}\omega_0\cos n\theta$$
$$u(\theta)=\sum_{n=1}^{\infty}u_0\sin n\theta \qquad (9)$$

After setting (8), (9) in equation (4) we obtain algebraic relations for the expansion coefficients ω_0, ω_1, ω_2, u_1, u_2, ω_n, u_n

$$\omega_0=-2(1+\nu)aT^*\varepsilon\left(\alpha_T^0+\varepsilon\left(\frac{\theta_0}{\pi}\right)\left(\frac{1}{2}-\frac{3}{4}\frac{\sin 2\theta_0}{\theta_0}+\cos^2\theta_0\right)\right)$$
$$\omega_1=0$$
$$u_1=0$$
$$\omega_2=\frac{2}{3}(1+\nu)aT^*\varepsilon\left(\frac{\theta_0}{\pi}\right)\left(\frac{1}{4}-\frac{1}{4}\frac{\sin 2\theta_0}{\theta_0}+\frac{1}{16}\frac{\sin 4\theta_0}{\theta_0}-\frac{\cos\theta_0\sin 3\theta_0}{3\theta_0}+\frac{\cos^2\theta_0\sin 2\theta_0}{2\theta_0}\right)$$
$$u_2=\frac{4}{3}(1+\nu)aT^*\varepsilon\left(\frac{1}{4}-\frac{1}{4}\frac{\sin 2\theta_0}{\theta_0}+\frac{1}{16}\frac{\sin 4\theta_0}{\theta_0}-\frac{\cos\theta_0\sin 3\theta_0}{3\theta_0}+\frac{\cos^2\theta_0\sin 2\theta_0}{2\theta_0}\right) \qquad (10)$$
$$u_n=(1+\nu)aT^*\frac{n}{(n^2-1)}F_n$$
$$\omega_n=(1+\nu)aT^*\frac{F_n}{(n^2-1)}$$

So the solution is given in the following form

$$u(\theta)=\frac{3}{4}(1+\nu)aT^*\varepsilon\left(\frac{\theta_0}{\pi}\right)\left(\frac{1}{4}-\frac{1}{4}\frac{\sin 2\theta_0}{\theta_0}+\frac{1}{16}\frac{\sin 4\theta_0}{\theta_0}-\frac{\cos\theta_0\sin 3\theta_0}{3\theta_0}+\right.$$
$$\left.+\frac{\cos^2\theta_0\sin 2\theta_0}{2\theta_0}\right)\sin 2\theta+2(1+\nu)aT^*\left(\frac{\varepsilon}{\pi}\right)\sum_{n=3}^{\infty}\frac{n}{(n^2-1)}F_n\sin n\theta$$
$$\omega(\theta)=-2(1+\nu)aT^*\left(\alpha_T^0+\varepsilon\left(\frac{\theta_0}{\pi}\right)\left(\frac{1}{2}-\frac{3}{4}\frac{\sin 2\theta_0}{\theta_0}+\cos^2\theta_0\right)\right)+ \qquad (11)$$
$$+\frac{2}{3}(1+\nu)aT^*\varepsilon\left(\frac{\theta_0}{\pi}\right)\left(\frac{1}{4}-\frac{1}{4}\frac{\sin 2\theta_0}{\theta_0}+\frac{1}{16}\frac{\sin 4\theta_0}{\theta_0}-\frac{\cos\theta_0\sin 3\theta_0}{3\theta_0}+\frac{\cos^2\theta_0\sin 2\theta_0}{2\theta_0}\right)\cos 2\theta+$$
$$+2(1+\nu)aT^*\left(\frac{\varepsilon}{\pi}\right)\sum_{n=3}^{\infty}\frac{n}{(n^2-1)}F_n\cos n\theta$$

Analysis of numerical results. In case where a coefficient of linear thermal expansion is given by means of the expression

$$\alpha_T^0=\begin{cases}\alpha_T^0, & \theta_0\le\theta\le\pi,\\ \alpha_T^o+\varepsilon(\cos\theta-\cos\theta_0)^2, & |\theta|\langle\theta_0, \quad \varepsilon\rangle 0\end{cases} \qquad (12)$$

let's consider the numerical model solution. At the same time calculations were carried out for cylindrical shell, which is characterized by the following output parameters: $\varepsilon = 0{,}01$, $\nu = 0{,}3$, $N = 100$, $\theta_0 = 0{,}05\pi$, $a = 1\,м$. Previous heat of the shell $T^* = 40^0$. The charts in figure 1 and figure 2 show the relevant dependence of the movements $u(\theta)$ and $\omega(\theta)$ on the changes of angular coordinate from $0 \le \theta \le \pi$.

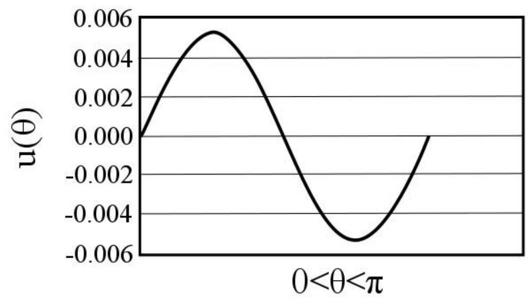

Figure 1. Dependence of the displacement $u(\theta)$ on the variation of the angular coordinate θ

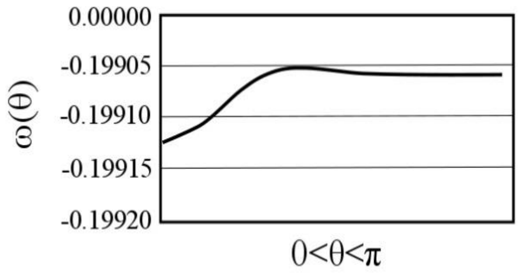

Figure 2. Dependence of the displacement $\omega(\theta)$ on the variation of the angular coordinate θ

Research results. From the graphs presented in Fig. 1 and Fig. 2 follows that the presence of heterogeneity according to thermal properties leads to displacements of circular shell of the median surface which confirms these theoretical calculations. The radial displacements thus are determined by the thermal deformation for $\alpha_T = const$. It should be mentioned that, the chosen approach and the results can determine the nature and location of the zones of heterogeneity. The results can be used in modern construction tasks, as well as in areas such as non-destructive testing in finding solutions of inverse problems of thermo mechanics.

Conclusions. Within the two-dimensional differential equations of thermo-elastic deformation of long cylindrical shell (plane problem) taking into account the heat distortion and dependence of coefficient of linear thermal expansion of angular coordinate θ in general and more specific cases, the setting is given and analytical solutions of the problem are got. With the use of this technique the expressions for the radial and circular displacements of the middle surface of the shell have been obtained. It is shown that the presence of heterogeneity on thermo physical properties that is described as dependent on the angular coordinate coefficient of linear thermal expansion α_T^0 can be found by careful measurement of radial and circular displacements of the points of the shell with a uniform heating to temperature $T^* = T - T_0 = const$. The developed approach and the results help to reveal not only the location of the damaged shell, but also to identify the size of area of heterogeneity. The results can be used in the design elements of the bearing shell structures.

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ВПЛИВ ТЕПЛОВОЇ НЕОДНОРІДНОСТІ НА ДЕФОРМУВАННЯ ТОНКОЇ КРУГОВОЇ ЦИЛІНДРИЧНОЇ ОБОЛОНКИ

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Резюме. *Розглянуто теплове деформування довгої циліндричної оболонки, яка знаходиться у зовнішньому температурному полі. Припущено, що матеріал оболонки має неоднорідності, які призводять до залежності коефіцієнта теплового розширення від кутової координати Ө. Для плоского випадку записано двовимірні диференційні рівняння термопружного деформування оболонки. Розв'язок задачі будується у вигляді рядів Фур'є. У випадку, коли коефіцієнт теплового розширення змінюється в діапазоні кутової координати, знайдено аналітичні вирази для радіальних і колових переміщень точок серединної поверхні. Виконано чисельні дослідження, які дозволяють визначити характер та розташування зони неоднорідності.*

Ключові слова: *теплова деформація, нелінійне деформування, ряди Фур'є, циліндрична оболонка.*

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