ФІНАНСИ. БАНКІВСЬКА СПРАВА

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EFFECT OF REINSURANCE ON THE PROBABILITY OF INSURANCE COMPANY BANKRUPTCY

Abstract. *In insurance theory and practice reinsurance is one of the most complex Business Intelligence decision-making procedures (BI-solutions), that is, operation between two insurance companies, in which one of them transmits and the other receives a part of the risk in exchange for insurance premium payment. Since the objectives of reinsurance decisions inevitably involve risk, so the formulation of relevant tasks should be to reduce this risk to a minimum. Therefore, a common approach to their development on the basis of risk theory and decision theory should include some probability assessment of parameter optimization. In the present article, based on stochastic modelling techniques the technique of reinsurance risk assessment is developed. An optimization criterion on the level of net profit under constraints on the probability of ruin is applied. The solution, which determines the parameters of the optimal strategy.*

Keywords: *insurance risks; reinsurance; stochastic modelling; underwriting.*

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ВПЛИВ ПЕРЕСТРАХУВАННЯ НА ЙМОВІРНІСТЬ БАНКРУТСТВА СТРАХОВОЇ КОМПАНІЇ

Анотація. *У теорії і практиці страхування однією з найбільш складних процедур прийняття Business Intelligence рішень (BI-рішень) є перестрахування, тобто операція між двома страховими компаніями, при якій одна з них передає, а інша приймає частину ризику в обмін на виплату страхової премії. Оскільки в задачах перестрахування прийняті рішення неминуче пов'язані з ризиком, то і постановка відповідних завдань повинна полягати в тому, щоб зводити цей ризик до мінімуму. Тому загальний підхід до їх розробки на основі теорії ризику та теорії прийняття рішень, повинен включати деякі ймовірні оцінки параметрів оптимізації. У представленій статті на підставі методів стохастичного моделювання розроблена методика оцінки ризиків перестрахування. Застосовані критерії оптимізації за рівнем чистого прибутку при обмеженнях на ймовірність збанкрутіння. Отримано рішення, що визначає параметри оптимальної стратегії.*

Ключові слова: *страхові ризики; перестрахування; стохастичні моделі; андерайтинг.*

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ВЛИЯНИЕ ПЕРЕСТРАХОВАНИЯ НА ВЕРОЯТНОСТЬ БАНКРОТСТВА СТРАХОВОЙ КОМПАНИИ

Аннотация. *В теории и практике страхования одной из наиболее сложных процедур принятия Business Intelligence решений (BI-решений) является перестрахование, т.е. операция между двумя страховыми компаниями, при которой одна из них передает, а другая принимает часть риска в обмен на выплату страховой премии. Поскольку в задачах перестрахования принимаемые решения неизбежно связаны с риском, то и постановка соответствующих* *задач должна заключаться в том, чтобы сводить этот риск к минимуму. Поэтому общий подход к их разработке на основе теории риска и теории принятия решений, должен включать некоторые вероятностные оценки параметров оптимизации. В представленной статье на основании методов стохастического моделирования разработана методика оценки рисков перестрахования. Применены критерии оптимизации по уровню чистой прибыли при ограничениях на вероятность разорения. Получено решение, определяющее параметры оптимальной стратегии.*

Ключевые слова: *страховые риски; перестрахование; стохастические модели; андеррайтинг.*

Urgency of the research. A question of ensuring the financial stability of the insurance companies is a complex one. Many solved problems in this case have mathematical nature. Number of authors developed mathematical methods that quantify the financial risks of the insurance company.

Even under conditions of economic stability insurance companies are not able to create a perfectly balanced portfolio of risks, since in most cases, the number of objects of insurance portfolio are insignificant or contains large and dangerous risks that contribute imbalances to the portfolio. Furthermore, experience shows that any insurance company, and in the case of a careful selection of risks when taking them to the insurance may not create a portfolio of these insurance objects fully isolated from each other, given that the insurance terms usually cover variety of risks that are insured objects may be exposed to, simultaneously upon the occurrence of disasters: floods, hurricanes, earthquakes, devastating fires, and so on. However, due to the fact that the financial resources, and even all of the assets of any insurer is only a small fraction of the total amount of his or her responsibility before insurers for the entire portfolio of insured objects, these insurance claims can not only considerably undermine the financial base of the insurance company, but also bring it to complete bankruptcy.

They resort to reinsurance when balancing insurance sums received on risk insurance, bringing the potential liability for the total sum insured and, accordingly, with financial possibilities of the insurer and to ensure the financial stability of insurance operations and their profitability, obtaining mutual participation in risks accepted for insurance by other insurers.

The need for reinsurance is particularly due to large risks the coverage of which one insurance company is not able to provide, as well as in connection with the possibility of accumulation of small risks.

The main purpose of reinsurance is risk dispersion that in turn, is based on the law of large numbers, which can be effectively applied to the probabilistic point of view only when the number of identical risks is maximum. In practice, no single company may have the identical number of risks and reinsurance task is to absorb the degree of fluctuation in the amount of losses resulting from inadequate use of large numbers law.

Actual scientific researches and issues analysis. The need to find new models of insurance risk management has recently received some coverage in papers. It should be noted that today, as a result of active risk management development the number of papers on various methods of risk assessment are increasing. We can note the works [1-3], where mathematic aspects of defining optimal reinsurance security structure. The most comprehensive results in this area obtained by the authors [4; 5], which focus on dynamic models of bankruptcy, different from the statistical fact that these events are considered for some time. However all the mentioned works cover only accident risk reduce, while other reinsurance risk components remain outside the scope of the study, and the task is solved only in a statistical statement. It should be noted that the study and generalization of the research results showed that at the moment there is no study on creation of an efficient and simple probabilistic method for estimating reinsurance risks affecting the insurer's financial stability, as well as a common methodological approach to risk determination, there are no general principles of risk assessment, there are unresolved questions about its measurement, selection and evaluation.

Uninvestigated parts of general matters defining. Currently insurance companies licensed for life insurance are facing the issue of reinsurance risks. For the European countries in life reinsurance the basic element in the calculation of the reinsurance premium is the insurance statistics of morbidity and mortality, and the risk is determined in a number of cases, the probability of death within a year. Nowadays, such detailed statistics in Ukraine is not conducted, and the available data provide an incorrect interpretation by using a low life expectancy of our population, thus arguing in favour of higher reinsurance rates. At present there are no clear mechanisms that link the mortality and morbidity statistics to the level of underwriting. It should also be noted that in this situation reinsurance companies unreasonably increase the reinsurance risk cost.

The **objective** of the paper is development of efficient and simple probabilistic risk assessment methods in reinsurance based on stochastic modelling.

The statement of basic materials. We consider the reinsurance contract under which a direct insurer will pay all losses to certain $infF = \alpha$, and the remaining amount covered by reinsurance company. From a mathematical point of view, this problem can be represented as follows:

$$Y_i = \begin{cases} Y, & \text{якшо } Y \le \alpha \\ \alpha, & \text{якшо } Y > \alpha \end{cases} = min(Y, \alpha)$$
$$Y_p = \begin{cases} 0, & \text{якшо } Y \le \alpha \\ Y - \alpha, & \text{якшо } Y > \alpha \end{cases} \tag{1}$$

where Y – the amount that must be paid to the insurer; and Y_i– part of the sum paid by the direct insurer $Y_i = h(Y) \le Y$, $Y_p = Y - Y_i$ – the differential, which reinsurance company has to pay. The parameter α is called the limit of the content, if this model is applied to each individual claim and the deductible in the case of the model description (1) of the general claim for a certain period of time.

The particular form of the function $h(Y)$ is determined by the terms and conditions of reinsurance. In the present model (1) $h(Y) = min(Y, \alpha)$

Due to the fact that the company's payouts, that transfers Y_i depends on the deductions α we introduce for this value indication $Y_i^{(\alpha)}$. Similarly, the share of which is to pay the reinsurance company will be denoted as $Y_p^{(\alpha)}$.

Given that direct insurer transfers part of the risk to the reinsurer, he pays him a certain reinsurance premiums, thus reducing their income. It should be noted that in general, it does not reduce the probability of bankruptcy P a company that sells. In accordance with the fact that re-insurance company assumes the risk of the company, gives its risks, for a premium, the reinsurance company for reinsurance process can be regarded as an ordinary insurance. İt should be noted that the reinsurance contracts are usually concluded at certain fixed points in time, therefore to describe the work of the company, that uses the services of reinsurer, it is advisable to use a discrete-time model, in particular the classical model of the sampled Cramer-Lundberg proposed by Uilmont [3]. To analyze this problem it is advisable to use the characteristic coefficient β (Lundberg coefficient). It should be noted that the coefficient β is the only positive root of the characteristic equation (2) with respect to Z.

$$\varphi(Z) = 1 + (1 + \theta)kZ, \tag{2}$$

where $k = EY$– the average size of insurance payments, $\varphi(Z) = Ee^{ZY}$ – a function that calculates the time of payment size; θ – relative insurance premium established before the insurance company. It is generally accepted [3] that the probability of bankruptcy of the insurance company is small, if the characteristic coefficient β is large, that is characteristic coefficient includes the basic model parameters (intensity of payments flow μ; the distribution of payments size Y, established before the insurance company; premiums received rate p).

After signing of reinsurance contract the intensity insurance claims stream μ will not change, but the size Y will look like $Y_i = h(Y)$, with the average amount of insurance payments

$$k_i = EY_i = Eh(Y) \tag{3}$$

When setting the problem, we assume that the distribution of the size of payments is described by an exponential law with parameter distribution μ, therefore using the characteristic equation (2), we obtain

$$p_i = (1+\theta)\mu k \text{ ,}$$

at the same time after reinsurance characteristic coefficient β for the direct insurer will be the only positive function root

$$P(Z) = Ee^{ZY_i} - 1 - \left[(1+\theta)k - (1+\theta_p)k_p\right]Z \tag{4}$$

If we assume that reinsurer uses the same relative load, and the company which transmits $\theta_p = \theta$, then (4) can be written as

$$P(Z) = Ee^{ZY_i} - 1 - (1+\theta)k_p Z \tag{5}$$

and the distribution function of the random variable $Y_i^{(\alpha)}$ will have the form:

$$F_i^{(\alpha)}(x) = P\left(Y_i^{(\alpha)} > x\right) = P\big(min(Y;\ \alpha)\big) = \begin{cases} P(Y > x), & \text{якщо } x < \alpha \\ 0, & \text{якщо } x \geq \alpha \end{cases} \tag{6}$$

We assume that the insurance company's portfolio consists of thousands of life insurance contracts for a period of 1 year. According to the terms of the contract the company pays a certain amount in case of death of the insured during the year and do not pay anything if the insured will live until the end of the year. We also assume that all insured the ones have the same probability of dying within a year, an equal q. Out of N thousands of insured ones N_1 an agreement is signed to the amount of b_1, N_2 - to the amount of b_2, N_3 - to the amount of b_3, N_4 - to the amount of b_4. The insurance company set relative insurance premium to the amount of θ. We assume that the company has signed an excessive loss reinsurance agreement with retention limits α. Provided that the re-insurance company sets its own rate based on the same mortality statistics as the previous company with the relative insurance premium, we define the way bankruptcy probability of the company changes, transmitting its risks, and its expected income. Taking into account (3) the bankruptcy probability and the expected income in the absence of reinsurance are determined by the mathematical expectation and variance of total payments and are equal to the sum of average values and variances of the sum of all individual losses.

$$EY = \sum_{s=1}^{N} EX_i\,, \qquad VarX = \sum_{i=1}^{N} VarX_i$$

Since the possible values of the payments under the individual contract take only two values, 0 with probability $1-q$ and b_i with probability q, we get

$$EX_i = qb_i, \qquad VarX_i = q(1-q)b_i^2 \tag{7}$$

It should be noted that calculating the values of (7) can be conveniently grouped into a size of insurance amount. Accordingly, the sum of the average values and variances of individual losses for contracts of k group are respectively equal to $N_k q b_k$ and $N_k q(1-q)b_k^2$. The total amount collected in the form of insurance premiums will be $u = (1+\theta)EY$, and the expected income of the company is $u - EY = \theta EY$. As stated in the work [2], the probability of bankruptcy P is determined from the following equation

$$P = (Y > u) = P\left(\frac{Y-EY}{\sqrt{VarY}} > \frac{u-EY}{\sqrt{VarY}}\right) \approx 1 - \Phi\left(\frac{u-EY}{\sqrt{VarY}}\right) \tag{8}$$

We assume that the insurance company has concluded a reinsurance agreement with a limit of detention α. The difference $EY - EY^{(\alpha)}$ The difference reflects the average total payout of the reinsurance company.. The assets of the company that sells the risks are reduced in this case from the value of the u to the value $u^{(\alpha)}$, and the expected revenue of the company, which transfers the risks is $u^{(\alpha)} - EY^{(\alpha)}$. To calculate the probability of bankruptcy, after reinsurance, we get

$$P^{(\alpha)} = P\left(Y^{(\alpha)} > u^{(\alpha)}\right) = P\left(\frac{Y^{(\alpha)} - EY^{(\alpha)}}{\sqrt{VarY^{(\alpha)}}} - \frac{u^{(\alpha)} - EY^{(\alpha)}}{\sqrt{VarY^{(\alpha)}}}\right) \approx \approx 1 - \Phi\left(\frac{u^{(\alpha)} - EY^{(\alpha)}}{\sqrt{VarY^{(\alpha)}}}\right) \quad (9)$$

A comparison of the values obtained by using the formulas (8) and (9), allows us to analyze how reinsurance affects the probability of bankruptcy of the company and find the value of the expected income. For example, an insurance company with N = 20 thousand contracts, N_1 = 10 thousand people, $b_1 = 100000$ грн, N_2 = 5000, $b_2 = 200000$ грн, N_3 =4000, $b_3 = 500000$ грн, N_4 = 1000, $b_4 = 1$ млн грн, with $q = 0{,}01$, relative insurance raise $\theta = 15\%$, $u = 500000$ грн and $\theta^* = 20\%$ considering the proposed technique, we find that reinsurance reduces the probability of bankruptcy from 5.82% to 4.85%. It should be noted that this result is achieved at the cost of reducing the expected revenue from 7500000 to 6510000.

Conclusions. In hypothesis that reinsurance process described by a sampled Cramer-Lundberg model the way reinsurance affects the insurance enterprise bankruptcy is analysed. Probabilistic method of calculating the expected income of the company is presented. It is shown that the case of the exponential distribution of insurance payments is the only one for which it is possible to solve the characteristic equation analytically. Equations defining the parameters of optimal strategies are found. Theoretical calculations are illustrated by numerical simulation.

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